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A pest management SI model with periodic biological and chemical control concern ☆

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Abstract

In this work, we consider an SI model for pest management, with concerns about impulsive releases of infective pests and pesticides sprays. We prove that all solutions of

$$\begin{cases} S'(t) = rS(t)\left(1 - \frac{S(t)+\theta I(t)}{K}\right) - \beta S(t)I^2(t), & t \neq n\tau, \\ I'(t) = \beta S(t)I^2(t) - wI(t), & t \neq n\tau, \\ \Delta S(t) = -\mu_1 S(t), & t = n\tau, \\ \Delta I(t) = -\mu_2 I(t) + \mu, & t = n\tau, \; n = 1, 2, \ldots, \end{cases} \tag{I}$$

are uniformly ultimately bounded and there exists globally asymptotic stability periodic solution of pest-extinction when $\ln\frac{1}{1-\mu_1} > r\tau - \frac{r\mu\theta(1-\exp(-w\tau))}{Kw(1-(1-\mu_2)\exp(-w\tau))} - \frac{\beta\mu^2(1-\exp(-2w\tau))}{2w(1-(1-\mu_2)\exp(-w\tau))^2}$ is satisfied, and the condition for permanence of system (I) is also obtained. It is concluded that the approach of combining impulsive infective releasing with impulsive pesticide spraying provides reliable tactic basis for practical pest management.

Keywords: Impulsive; Infective; Chemical control; Pest- extinction

1. Introduction

According to reports of the Food and Agriculture Organization, the warfare between man and pests has sustained for thousands of years. With the development of society and progress of science and technology, human have adopted some advanced and modern weapons, for instance chemical pesticides, biological pesticides, remote sensing and measure, computers, atomic energy etc., where some brilliant achievements have

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been obtained. However, the warfare is not over, and will continue. A great deal of and a large variety of pesticides were used to control pests. In all, pesticides are useful because they can quickly kill a significant portion of a pest population and sometimes provide the only feasible method for preventing economic loss. However, pesticide pollution is also recognized as a major health hazard to human beings and beneficial insects. Lengthening the period of pesticides spraying may reduce the cost of pest management. In this paper, a good approach is given which combines pesticides efficacy tests with biological research.

The use of bacteria, fungi and viruses is potentially one of the most important approaches in pest control. For example, *Bacillus thuringiensis*, which is available in commercial preparations, is used in the control of a large number of pests [1–3,8,9]. An advantage of using insect pathogens is that they are safe to man and are usually safe to beneficial insects.

There is a vast amount of literature on the applications of microbial disease to suppress pests [3,4,7,9,10], and many good articles [15–25] devote to disease transmission, but there are a few papers on a mathematical model of the dynamical of microbial disease in pest control [5,6,11,31]. In this paper, we will introduce additional infective pests, which are obtained in the laboratory, into a natural SI system with spaying pesticides for pest control. We shall examine the strategy of combining periodic releasing of infective pests with periodic spraying of pesticides in a more flexible manner.

2. Model formulation

The basic SI model is

$$\begin{cases} S'(t) = -\beta S(t)I(t), \\ I'(t) = \beta S(t)I(t) - wI(t), \end{cases} \tag{2.1}$$

where $\beta > 0$ is called the transmission coefficient, $w > 0$ is called the death coefficient of $I(t)$, $S(t)$ denotes the number of susceptible insects and $I(t)$ denotes the number of infective insects. In our models we assume that all newborns are susceptible, and the basic model considered in [12] follows a model of the epidemic under a control variable:

$$\begin{cases} S'(t) = -\beta S(t)I(t), \\ I'(t) = \beta S(t)I(t) - wI(t) + u(t), \end{cases} \tag{2.2}$$

where $u(t)$, which is a control variable, denotes the rate of pests infected in the laboratory, and there are some other conditions for the above system, but we consider that the variable $u(t)$ is difficult in practice. And the susceptible pests $S(t)$ will not go to extinction with regard to human beings and some mass residing animals, Anderson et al. pointed out that standard incidence is more suitable than bilinear incidence [26–28]. Levin et al. have adopted a incidence form like $\beta S^q I^p$ or $\frac{\beta S^q I^p}{N}$ which depends on different infective diseases and environments [29,30]. So we develop (2.2) by introducing a constant periodic releasing of the infective pests and spaying pesticides at a fixed moment, that is, we consider the following impulsive differential equation with a fixed moment:

$$\begin{cases} S'(t) = rS(t)\left(1 - \frac{S(t)+\theta I(t)}{K}\right) - \beta S(t)I^2(t), & t \neq n\tau, \\ I'(t) = \beta S(t)I^2(t) - wI(t), & t \neq n\tau, \\ \Delta S(t) = -\mu_1 S(t), & t = n\tau, \\ \Delta I(t) = -\mu_2 I(t) + \mu, & t = n\tau, \quad n = 1, 2, \ldots, \end{cases} \tag{2.3}$$

where $r > 0$ is the intrinsic growth rate of pests, $K > 0$ is the pests capacity of environment, $\Delta I(t) = I(t^+) - I(t)$, $0 < \theta < 1$, $\mu \geqslant 0$ is the released amount of infective pests at $t = n\tau$, $n \in Z_+$, $1 > \mu_1 \geqslant 0$, $1 > \mu_2 \geqslant 0$ respectively, which represents the portion of susceptible and infective pests due to spraying pesticides at $t = n\tau$, $n \in Z_+$ and $Z_+ = \{1, 2, \ldots\}$, τ is the period of the impulsive effect, that is, we can use a combination of biological and chemical tactics to eradicate pests or keep the pest population below the damage level.