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Fuzzy regression based on asymmetric support vector machines ☆

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Abstract

This paper presents a modified framework of support vector machines which is called asymmetric support vector machines (ASVMs) and is designed to evaluate the functional relationship for fuzzy linear and nonlinear regression models. In earlier works, in order to cope with different types of input–output patterns, strong assumptions were made regarding linear fuzzy regression models with symmetric and asymmetric triangular fuzzy coefficients. Excellent performance is achieved on some linear fuzzy regression models. However, the nonlinear fuzzy regression model has received relatively little attention, because such nonlinear fuzzy regression models having certain limitations. This study modifies the framework of support vector machines in order to overcome these limitations. The principle of ASVMs is applying an orthogonal vector into the weight vector in order to rotate the support hyperplanes. The prime merits of the proposed model are in its simplicity, understandability and effectiveness. Consequently, experimental results and comparisons are given to demonstrate that the basic idea underlying ASVMs can be effectively used for parameter estimation.

Keywords: SVMs; Fuzzy regression; Nonlinear; Asymmetric; Orthogonal vector

1. Introduction

Fuzzy regression, a fuzzy type of classical regression analysis, was proposed to evaluate the functional relationship between dependent and independent variables in a fuzzy environment [12]. Since Tanaka et al. introduced linear fuzzy regression, this subject has been extensively studied and successfully applied to various fields such as forecasting, system identification, and others [8,11,14].

Generally speaking, two different forms of linear fuzzy regression models are used by most studies. These two forms are: the approach of Tanaka, which minimizes the total spread of the output, and the fuzzy least square method, which minimizes the total square error of the output [15,21]. Moreover, symmetric triangular

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fuzzy coefficients are most frequently used in linear fuzzy regression models. However, such symmetric fuzzy regression methods have some limitations, and as a result the solution they provide is not optimal in most case, even if it has excellent performance. Tanaka showed that better solution is reached when non-symmetric triangular fuzzy coefficients are adopted on linear fuzzy regression model [13].

Although the linear fuzzy regression model has been extensively studied, nonlinear fuzzy regression has received little attention [2,5,9,10]. Ishibuchi proposed an asymmetric fuzzy regression model, based on the neural network, to overcome the limitations pointed out in the above paragraph. Ishibuchi's model employed a multilayer feedforward neural network to model the nonlinear mapping. However, the performance of his model provides only a minor improvement, because it cannot overcome the inherent limitations of applying the back-propagation learning algorithm to training MLPs.

In another study, Hong and Hwang used support vector machines (SVMs) to design a linear and nonlinear fuzzy regression model [9]. SVMs were introduced to solve pattern recognition and regression problems [3,7,19,20]. According to Vapnik [20], the advantage of SVMs is that the risk of misclassifying is minimized, not only for the examples in the training set, but also for the unseen examples of the test set. Another main advantage of the SVMs is that its performance and complexity are independent of the dimensionality of the input domain [16,19,20]. It is evident that there are significant advantages which can be achieved by introducing SVMs into the fuzzy regression model. When the fuzzy regression model is transferred to consider the interval regression model, optimal solution is determined by minimizing the total widths of the estimated interval outputs. As we know, the intervals are denoted by lower limit, upper limit and center. On multi-dimensional space, lower limit and upper limit can be taken as hyperplane, so that the linear fuzzy regression problem can be formulated as minimizing the margin which is bounded by these two hyperplanes. On the other hand, on SVMs the optimal separating hyperplane is the maximum margin hyperplane with the geometric margin $\frac{2}{\|w\|}$ which can be obtained by minimizing the cost function $\frac{1}{2}\|w\|^2$. Intuitively, the margin of separation can be treated as the region which is bounded by the upper limit and the lower limit while SVMs are introduced into the fuzzy regression model. Hence, the minimal margin on fuzzy regression model should be obtained by maximizing the cost function $\Phi(w) = \frac{1}{2}\|w\|_2^2$. However, this solution is hard to arrive at by running linear programming. Since it is so difficult to solve, another technique needs to be adopted, such as the ε-loss function [19,20]. However, minimizing the cost function $\Phi(w)$ is still included in order to reduce model complexity such that the conflict of cost function does exist. Although Hong and Hwang proposed a new method for introducing the use of SVMs for the fuzzy regression model and their method provided excellent performance, their model did not overcome the conflict.

This paper proposes a modified framework for the support vector machines, called the asymmetric support vector machines (ASVMs), to overcome the difficulties mentioned earlier. On asymmetric support vector machines, two support hyperplanes, representing the borders of margin of separation are constructed which are different to SVMs. The idea of asymmetric support vector machines originates from joining an orthogonal vector into the weight vector in order to rotate the support hyperplanes. With this modification, the support hyperplanes can fit the borders of optimal margin of separation. With this new architecture, ASVMs are adaptable to any cases and provide excellent performance for the fuzzy regression problem.

The remainder of this paper is organized as follows. Section 2 reviews the basic concept of the fuzzy regression model and SVMs. Section 3 then introduces the basic idea of ASVMs. Next, Section 4 represents the architecture of ASVMs. Section 5 conducts the model of linear and nonlinear fuzzy regression based on ASVMs. Finally, Section 6 discusses some experimental results.

2. Basic concepts

2.1. Linear fuzzy regression model

Let $F(\cdot)$ denote a mapping from R^n to $\widetilde{R}$, $z = (1, z_1, z_2, \ldots, z_n)$, and Y be a normal fuzzy number, respectively, where $\widetilde{R}$ is the set of fuzzy numbers. Then, the linear fuzzy regression model can be represented as follows:

$$Y = F(z) = A_0 + A_1 z_1 + A_2 z_2 + \cdots + A_n z_n, \tag{1}$$

where A_p, $p = 0, 1, \ldots, n$, are fuzzy coefficients presented in the form of symmetric triangular fuzzy numbers.