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Selecting the optimum portfolio using fuzzy compromise programming and Sharpe's single-index model

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Abstract

Different approaches besides the traditional Markowitz's model have been proposed in the literature to analyze portfolio selection problems. Among them, Compromise Programming (CP) is a suitable multiobjective programming technique which allows the handling of several objectives in those situations in which the existence of a high level of conflict between criteria does not permit the simultaneous optimization of all the considered objectives.

When objectives and constraints are in an imprecise environment Fuzzy CP arises as a suitable solving method. Imprecision will be quantified by means of fuzzy numbers that represent the continuous possibility distributions for fuzzy parameters and hence place a constraint on the possible values the parameters may assume.

In this paper a new Fuzzy Compromise Programming approach is proposed based on the obtaining of the minimum fuzzy distance to the fuzzy ideal solution of the portfolio selection problem. Once this fuzzy distance has been obtained the second step consists of finding a crisp decision vector, an optimal portfolio, implying a fuzzy distance to the ideal solution the more accurate as possible to the fuzzy minimum distance previously obtained.

Keywords: Portfolio selection; Sharpe's single-index model; Fuzzy numbers; Value, fuzziness and ambiguity; Fuzzy compromise programming

1. Introduction

In the portfolio selection problem, given a set of available assets, the objective consists of finding out the optimum way of investing a particular budget in these assets. Each different way to diversify the investor's budget between the considered assets is called portfolio.

Analysts intend to forecast the performance of the market but given the inherent uncertainty in financial markets they are very cautious in expressing their guesses. Fuzzy sets theory and possibility theory appear to be useful tools to solve problems in uncertain and imprecise environments where these questions arise and, hence, have been widely and successfully used (e.g. [1,2]). In fact, there exist a lot of published works in the field of Finance which try to incorporate fuzzy sets theory as a helpful tool to deal with uncertainty

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(see [3–12] as examples). In this paper the portfolio selection problem has been also handled in a soft framework by means of a flexible model for decision making designed including imprecise expert knowledge from analysts and investors. In this sense, an extension of Sharpe's single-index model including fuzzy parameters is proposed. The use of this model requires beta estimation for each asset that is a potential candidate to be included in a portfolio (see [10]). Therefore, in this paper, in order to determine portfolio's risk and return, subjective estimations will be included in the model described in fuzzy terms by means of fuzzy numbers.

Within the multiple criteria decision making field compromise programming (CP) [13,14] arises as a useful tool to deal with the portfolio selection problem. CP is a mathematical programming technique with the capability of handling multiple objectives in those situations where the existence of a high level of conflict between criteria does not allow the simultaneous optimization of all the considered objectives. In those situations it seems rational to find compromise solutions between the level of achievement of the objectives. One of the pioneering applications of CP for portfolio selection are due to Ballestero and Romero [15] and since then several interesting works can be found in the literature (see [16–18,9,19,20] and [12] among others).

Many fuzzy multiobjective programming approaches incorporate defuzzification processes which map fuzzy values into crisp ones. Defuzzification takes all this away since it reduces a fuzzy value into one or more single crisp values and so it can be described as a process of synthesis. As such, the whole concept of defuzzification is completely opposite to the main purpose of fuzzy sets theory and therefore, maintenance of fuzziness into the last steps of the resolution process is a desirable feature.

Designing a Fuzzy CP model involves the minimization of the *distance* between fuzzy numbers, the fuzzy ideal solution and the fuzzy objectives. Arenas et al. [21] and Bilbao et al. [9] have proposed methods for dealing with fuzzy CP, which handle fuzzy numbers through their expected intervals and they introduce the discrepancy concept into the model as defuzzification method. In this paper a new fuzzy CP approach is proposed to solve a real portfolio selection problem trying to maintain the imprecision and/or subjectivity inherent to some data in the model until the last phases of the resolution. The proposed method relies in two steps. In the first step the minimum fuzzy distance to the fuzzy ideal solution is obtained. Once this fuzzy distance has been obtained the second step of the model consists of finding a crisp decision vector, an optimal portfolio implying a fuzzy distance to the ideal solution the more accurate as possible to the fuzzy minimum distance previously obtained.

The paper is organized as follows. In Section 2 the portfolio selection model proposed in this paper is presented. Section 3 proposes a new approach of Fuzzy CP to solve the portfolio selection problem. In Section 4, in order to illustrate the suitability and applicability of the proposed model a real portfolio selection problem is presented. Thirty one Spanish mutual funds of the type called *Fondos de Inversión Mobiliaria* (FIM) have been considered, whose quarterly returns data are referred to the period 1999–2003 and an optimum fuzzy compromise portfolio have been obtained. In the last section the main conclusions of the paper are presented.

2. Portfolio selection using Sharpe's single-index model

The mean-variance model for the portfolio selection problem first proposed by Markowitz in 1959 [22], has played an important role in the development of portfolio selection modern theory. Suppose that $P = (x_1, x_2, \ldots, x_n)$ is a portfolio, consisting of n assets $s_1, s_2, \ldots, s_n$. The Markowitz's mean-variance model for portfolio selection minimizes risk while achieving a predetermined level of expected return, E_0. The risk of each asset is measured by the variance of its return. If each component x_i of the n-vector x represents the proportion of an investors' wealth allocated to asset i, then the return of the portfolio is given by the scalar product of x by the vector of individual asset returns. Therefore, if $R = (R_1, R_2, \ldots, R_n)$ denotes the n-vector of returns of the assets and C the $n \times n$ covariance matrix of the returns, we obtain the expected portfolio return by the expression $\sum_{i=1}^{n} E(R_i)x_i$ and its level of risk by $\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_ix_j$. Analytically, the problem can be formulated as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \sigma_{ij} x_i x_j \\ \text{s.t.} \quad & \sum_{i=1}^{n} E(R_i) x_i \geqslant E_0, \end{aligned}$$