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Solving a linear programming problem with the convex combination of the max–min and the max-average fuzzy relation equations

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Abstract

In this paper, we introduce a fuzzy operator constructed by the convex combination of two known operators, max–min and max-average compositions [H.J. Zimmermann, Fuzzy set theory and it's application, Kluwer Academic Publishers, Boston, Dordrecht, London, 1999]. This operator contains some properties of the two known compositions when it generates the feasible region for linear optimization problems. We investigate linear optimization problems whose feasible region is the fuzzy sets defined with this operator. Thus, firstly, the structure of these fuzzy regions is considered and then a method to solve the linear optimization problems with fuzzy equation constraints regarding this operator is presented.
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Keywords: Linear objective function optimization; Fuzzy relation equations; Fuzzy relation compositions

1. Introduction

Let $I = \{1, 2, \ldots, m\}$ and $J = \{1, 2, \ldots, n\}$. A fuzzy relation equation can be defined as follows:

$$x \oplus A = b, \tag{1}$$

where $A = (a_{ij})_{m\times n}$ is a fuzzy matrix and $b = (b_j)$ is an n-dimensional vector such that $0 \leqslant a_{ij} \leqslant 1$ and $0 \leqslant b_j \leqslant 1$, $\forall i \in I$ and $\forall j \in J$. Moreover, the operator "$\oplus$" is a linear convex combination of max–min and max-average compositions [1] which is defined as follows:

$$\max_{i\in I}\left\{\lambda \min\{x_i, a_{ij}\} + (1-\lambda)\frac{x_i + a_{ij}}{2}\right\} = b_j \quad \forall j \in J,\ \ \lambda \in [0, 1]. \tag{2}$$

At first, our purpose is to find an m-dimensional vector $x = (x_1, x_2, \ldots, x_m)$ satisfying $x \oplus A = b$ such that $0 \leqslant x_i \leqslant 1$, $\forall i \in I$. This object will lead to the characterization and determination of the feasible region of

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Eq. (1). Then, we will study the linear programming problems subject to the fuzzy relation equations as the feasible region defined as (1). Actually, these problems can be generally considered as given below:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} c_i x_i \\ & x \oplus A = b, \\ & 0 \leqslant x_i \leqslant 1 \quad \forall i \in I, \end{aligned} \tag{3}$$

where $c = (c_1, c_2, \ldots, c_m)$ is an m-dimensional vector in which c_i is the cost coefficient associated with variable x_i.

The resolution of the fuzzy relation equations is an interesting and considerable problem investigated by many researchers [2–5,8–16]. Eq. (1) with max–min composition are often studied. Truly, the feasible region of Eq. (1) with max–min composition is a non-convex set determined in terms of one maximum solution and the finite number of the minimum solutions [3,8]. Recently, the linear programming problem (3) with max–min composition has been studied by Fang and Li who have presented a branch and bound method to solve it [7]. On the other hand, the non-convex feasible region of Eq. (1) with max-average composition, the determination of this region in terms of one maximum solution and the finite number of the minimum solutions and the resolution of the problem (3) with this composition have been investigated by Khorram and Ghodousian who have given a tabular method to optimize such problems [17]. Unlike the regular linear programming problems [6], the non-convexity property of the feasible region is one of the ordinary characteristics in such problems which prohibit us to apply traditional methods such as simplex and interior point for optimizing.

We will show that many of the feasible region characteristics of Eq. (1) with the operator defined in (2) coincide with that of Eq. (1) concerning to max-average and max–min compositions. Moreover, we will show the tabular method applied to optimize problem (3) with max-average composition can be used again to solve the same problems with "$\oplus$" operator. From (2) it is obvious that if $\lambda = 1$ then operator "$\oplus$" is turned into max–min composition and if $\lambda = 0$ then it turned into max-average composition. Therefore, we analyze (2), when $\lambda \in (0,1)$, so as to study this combined operator. The fuzzy relation equations have the critical role in the fuzzy modeling, fuzzy diagnosis and fuzzy control and also they serve important applications in fields such as psychology, medicine, economics and sociology [4,18].

In this paper, in Section 2, we analyze the feasible region of Eq. (1) and its characteristics. In Section 3, some properties of the feasible solutions are derived that are important and primary for us in order to apply tabular method for solving problem (3). In Section 4, the tabular method is presented and in Section 5, we give an example to illustrate the method.

2. The feasible region characterization

Let $X = \{x \in R^m : 0 \leqslant x_i \leqslant 1, \forall i \in I\}$. The feasible region of the problem (3) is denoted by $X[A,b] = \{x \in X : x \oplus A = b\}$. We remember that for each ${}^1x, {}^2x \in X$, it is said to be ${}^1x \leqslant {}^2x$ iff ${}^1x_i \leqslant {}^2x_i$, $\forall i \in I$. It is obvious that $\leqslant$ is a partial ordering on X.

Moreover, $\hat{x} \in X[A,b]$ is said to be the maximum solution if $x \leqslant \hat{x}$, $\forall x \in X[A,b]$. Similarly, $\overset{\vee}{x} \in X[A,b]$ is called a minimum solution if $x \leqslant \overset{\vee}{x}$ implies $x = \overset{\vee}{x}$, $\forall x \in X[A,b]$.

To determine the set $X[A,b]$, we firstly decompose Eq. (1) into the equations below

$$x \oplus a_j = b_j \quad j = 1, 2, \ldots, n, \tag{4}$$

where a_j is j the column of the matrix A. Let ${}^jX[A,b]$ denote the feasible region of Eq. (4) for a fixed $j \in J$ which is defined as ${}^jX[A,b] = \{x \in X : x \oplus a_j = b_j\}$.

From (2) and (4) it is obvious that $x \in {}^jX[A,b]$ iff $x \in X$ and

$$\begin{aligned} &\text{(a)} \quad \lambda \min\{x_i, a_{ij}\} + \frac{1}{2}(1-\lambda)(x_i + a_{ij}) \leqslant b_j \quad \forall i \in I, \\ &\text{(b)} \quad \lambda \min\{x_i, a_{ij}\} + \frac{1}{2}(1-\lambda)(x_i + a_{ij}) = b_j \quad \exists i \in I. \end{aligned} \tag{5}$$

We can easily attain two below useful lemmas that are very useful in some next theorems proof.

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