Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Applied Mathematics and Computation 179 (2006) 92–97
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Rectangular decomposition method for fractional diffusion-wave equations

Zaid M. Odibat

Prince Abdullah Bin Ghazi Faculty of Science & IT, Al-Balqa' Applied University, Salt, Jordan

Abstract

In this paper, a new modification of the Adomian decomposition method is effectively implemented for solving fractional diffusion-wave equations that will facilitate the calculations, where the fractional derivative is based on Caputo definition. The proposed algorithm gives an analytical solution in the form of a convergent series with easily computable components. Numerical examples are given to show the application of the present algorithm. The new modification introduces a promising tool for many partial differential equations.

Keywords: Rectangular decomposition method; Fractional integral; Caputo derivative; Diffusion-wave equation

1. Introduction

The Adomian decomposition method, which was develop by Adomian (see [8]), has been receiving much attention in recent years in the area of series solution. The method has a significant advantage over numerical methods in that it provides analytic solution in a rapid convergent series with elegantly computable components. This method is applied recently to a wide class of linear and nonlinear, ordinary and partial differential equations, stochastics and deterministic problems in many mathematics and physics areas. For more details about the method and its applications see [3,4,8,11–13].

Fractional differential equations have been the focus of many studies due to their frequent appearance in various fields such as physics, chemistry and engineering [1,6,9,10,15,18]. The space–time diffusion and wave equations are mathematical models of a wide range of physical phenomena. They have been the subject of many papers by Schneider and Wyss [20], Fujita [22], Gorenflo and Mainardi [16], Luchko and Gorenflo [23], Metzler and Klafter [17], Mainardi [5], Mainardi et al. [7], Agrawal [14], Hanyga [2] and others.

The space–time-fractional diffusion-wave equation

$$D_t^{\alpha}u(x,t) = \lambda D_x^{\beta}u(x,t), \quad t > 0,\ \ x > 0,\ \ \lambda > 0 \tag{1.1}$$

E-mail address: odibat@bau.edeu.jo

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.11.088

is obtained by replacing the time derivative in the ordinary diffusion-wave equation by a fractional derivative of order α, $0 < \alpha \leqslant 2$, and replacing the space derivative by a fractional derivative of order β, $0 < \beta \leqslant 2$ (the range $0 < \alpha \leqslant 1$ corresponds to the fractional diffusion equation, while the range $1 < \alpha \leqslant 2$ corresponds to the fractional wave equation). Fujita [21] presented the existence and uniqueness of the solution of the Cauchy problem of the type

$$D_t^{\alpha}u(x,t) = D_x^{\beta}u(x,t), \quad \alpha \geqslant 0, \ \beta \leqslant 2. \tag{1.2}$$

It is the purpose of this paper to establish a new reliable algorithm of Adomian decomposition method, called the rectangular decomposition method, that will be proposed in Section 2. The new modification will be used to solve the fractional partial differential equation (1.1) in terms of rapid convergent series.

Now we will introduce the following definitions and properties of fractional integral and Caputo fractional derivative.

1.1. The fractional integral

According to the Riemman–Liouville approach to fractional calculus, the fractional integral of order $\alpha > 0$ is defined as

$$J^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau, \tag{1.3}$$

$$J^0 f(t) = f(t). \tag{1.4}$$

Details and properties of the operator J^{α} can be found in [18], we mention the following:

For α, $\beta > 0$, $t > 0$ and $\gamma > -1$, we have

$$J^{\alpha}J^{\beta}f(t) = J^{(\alpha+\beta)}f(t), \tag{1.5}$$

$$J^{\alpha}J^{\beta}f(t) = J^{\beta}J^{\alpha}f(t), \tag{1.6}$$

and

$$J^{\alpha}t^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1+\alpha)}t^{\gamma+\alpha}. \tag{1.7}$$

1.2. The Caputo fractional derivative

Let m be the smallest integer that exceeds α, then the Caputo fractional derivative of order $\alpha > 0$ is defined as

$$D^{\alpha}f(t) = J^{(m-\alpha)}[f^{(m)}(t)], \tag{1.8}$$

namely

$$D^{\alpha}f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\left[\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}}\,\mathrm{d}\tau\right], & m-1 < \alpha < m, \\ \frac{\mathrm{d}^m}{\mathrm{d}t^m}f(t), & \alpha = m. \end{cases} \tag{1.9}$$

The Caputo fractional derivative was investigated by many authors, see [15], its considered here because it allows traditional initial and boundary conditions to be included in the formulation of the problem. The following are some of its basic properties, for $m-1 < \alpha \leqslant m$,

$$D^{\alpha}\left(\sum_{j=1}^{m} t^{m-j}\right) = 0, \tag{1.10}$$

$$D^{\beta}J^{\alpha} = J^{\alpha-\beta}, \quad \beta \leqslant \alpha, \tag{1.11}$$