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A bivariate rational interpolation and the properties

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Abstract

In this paper a bivariate rational interpolation is constructed using both function values and partial derivatives of the function being interpolated as the interpolation data. The interpolation function has a simple and explicit rational mathematical representation with parameters, and it can be expressed by the symmetric bases. It is proved that the interpolation is stable. The concept of integral weights coefficients of the interpolation is given, which describes the "weight" of the interpolation points and the quantity as the interpolation data in the local interpolating region.

Keywords: Bivariate interpolation; Rational spline; Computer-aided geometric design

1. Introduction

The construction method of the curve and surface and the mathematical description of them is a key issue in computer-aided geometric design. There are many ways to tackle this problem [1–10], for example, the polynomial spline method, the NURBS method and the Bézier method. These methods are effective and applied widely in shape design of industrial products. Generally speaking, most of the polynomial spline methods are the interpolating methods, which means that the curves or surfaces constructed by the methods pass through the interpolating points. The NURBS and Bézier methods are the so-called "no-interpolating type" methods; this means that the constructed curve and surface do not pass through the given data, and the given points play the role of the control points. In general, therefore, the interpolation method gives a better approximation to the function being approximated than that constructed by the other methods.

In recent years, the univariate rational spline interpolation with parameters has been constructed [11–19]. Those kinds of interpolation spline not only have simple mathematical representation, they can be used for the modification of local curves by selecting suitable parameters under the condition that the interpolating data are not changed. The bivariate spline interpolation method does not appear very often in the literature. Indeed, so far there are few such bivariate interpolating splines which have simple and explicit mathematical

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representation and can be modified by the parameters. Motivated by the univariate rational spline interpolation, the bivariate rational interpolation with parameters, based only on the values of the function being interpolated, has been studied in [20]. This paper will deal with the bivariate interpolation based on both function values and partial derivative values of the function being interpolated.

The paper is arranged as follows. In Section 2, the new bivariate rational spline based on function values and partial derivatives with parameters is constructed. More important is that the symmetric bases of this bivariate interpolation can be derived from the process of its construction, they are discussed in Section 3. Section 4 is about some properties of the interpolation, the concept of integral weights coefficients of the interpolation is given, which describes the "weight" of the interpolation points and interpolating quantities in the local interpolating region. Section 5 is about the stability of the interpolation, it have been proved that the values of the interpolation function must be in an interval related to the interpolating data no matter what the positive parameters might be. An example is given in Section 6, which shows that this interpolation is with good approximation to the function being interpolated.

2. Interpolation

Let $\Omega:[a,b;c,d]$ be the plane region, and $\left\{\left(x_i, y_j, f_{i,j}, \frac{\partial f_{i,j}}{\partial x}, \frac{\partial f_{i,j}}{\partial y}\right),\ i=1,2,\ldots,n;\ j=1,2,\ldots,m\right\}$ be a given set of data points, where $a=x_1<x_2<\cdots<x_n=b$ and $c=y_1<y_2<\cdots<y_m=d$ are the knot spacings, $f_{i,j}$, $\frac{\partial f_{i,j}}{\partial x}$, $\frac{\partial f_{i,j}}{\partial y}$ represent $f(x_i,y_j)$, $\frac{\partial f(x,y)}{\partial x}$, $\frac{\partial f(x,y)}{\partial y}$ at the point (x_i,y_j) respectively. Let $h_i=x_{i+1}-x_i$, $l_j=y_{j+1}-y_j$, and for any point $(x,y)\in[x_i,x_{i+1};y_j,y_{j+1}]$ in the (x,y)-plane, and let $\theta=\frac{x-x_i}{h_i}$ and $\eta=\frac{y-y_j}{l_j}$. First, for each $y=y_j$, $j=1,2,\ldots,m$, construct the x-direct interpolating curve $P^*_{i,j}(x)$ in $[x_i,x_{i+1}]$ [18]; this is given by

$$P^*_{i,j}(x)=\frac{p^*_{i,j}(x)}{q^*_{i,j}(x)},\quad i=1,2,\ldots,n-1, \tag{1}$$

where

$$p^*_{i,j}(x)=(1-\theta)^3\alpha^*_{i,j}f_{i,j}+\theta(1-\theta)^2V^*_{i,j}+\theta^2(1-\theta)W^*_{i,j}+\theta^3\beta^*_{i,j}f_{i+1,j},$$
$$q^*_{i,j}(x)=(1-\theta)\alpha^*_{i,j}+\theta\beta^*_{i,j},$$

and

$$V^*_{i,j}=(2\alpha^*_{i,j}+\beta^*_{i,j})f_{i,j}+h_i\alpha^*_{i,j}\frac{\partial f_{i,j}}{\partial x},$$
$$W^*_{i,j}=(\alpha^*_{i,j}+2\beta^*_{i,j})f_{i+1,j}-h_i\beta^*_{i,j}\frac{\partial f_{i+1,j}}{\partial x},$$

with $\alpha^*_{i,j}>0$, $\beta^*_{i,j}>0$. This interpolation is called the rational cubic interpolation based on function values and derivatives which satisfies

$$P^*_{i,j}(x_i)=f_{i,j},\quad P^*_{i,j}(x_{i+1})=f_{i+1,j},\quad P^{*\prime}_{i,j}(x_i)=\frac{\partial f_{i,j}}{\partial x},\quad P^{*\prime}_{i,j}(x_{i+1})=\frac{\partial f_{i+1,j}}{\partial x}.$$

Obviously, the interpolating function $P^*_{i,j}(x)$ on $[x_i,x_{i+1}]$ is unique for the given data $\left\{x_r, f_{r,j}, \frac{\partial f_{r,j}}{\partial x}, r=i,i+1\right\}$ and positive parameters $\alpha^*_{i,j}$, $\beta^*_{i,j}$.

Using the x-direction interpolation function, $P^*_{i,j}(x)$, $i=1,2,\ldots,n-1$; $j=1,2,\ldots,m$ defines the bivariate rational interpolating function in $[x_1,x_n;y_1,y_m]$. For each pair (i,j), $i=1,2,\ldots,n-1$ and $j=1,2,\ldots,m-1$, let $\alpha_{i,j}>0$, $\beta_{i,j}>0$, define the bivariate interpolating function $P_{i,j}(x,y)$ on $[x_i,x_{i+1};y_j,y_{j+1}]$ as follows:

$$P_{i,j}(x,y)=\frac{p_{i,j}(x,y)}{q_{i,j}(y)},\quad i=1,2,\ldots,n-1;\ j=1,2,\ldots,m-1, \tag{2}$$

where

$$p_{i,j}(x,y)=(1-\eta)^3\alpha_{i,j}P^*_{i,j}(x)+\eta(1-\eta)^2V_{i,j}+\eta^2(1-\eta)W_{i,j}+\eta^3\beta_{i,j}P^*_{i,j+1}(x),$$
$$q_{i,j}(y)=(1-\eta)\alpha_{i,j}+\eta\beta_{i,j},$$