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Computational comparison for ML estimator of quadratic functions of the Bernoulli parameter in IS and FSS methods

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Abstract

In this paper, we suggest the maximum likelihood estimator (MLE) of the quadratic function $p^2 + \beta p$ (for different values of β) of the Bernoulli parameter, that is discussed for the inverse sampling (IS) and fixed size sampling (FSS) methods. Moreover, the IS method is compared with FSS method based on mean squared error (MSE). In consequence of MSE for MLE of the quadratic function $p^2 + \beta p$ in two sampling methods are complicated, so we also give a computational comparison of MSE for $p^2 + \beta p$ to assess the performance of two sampling methods by using numerical method.

Keywords: Mean squared error; Biased estimator; Fixed size sample; Inverse sample; Maximum likelihood estimator; Bernoulli parameters; Hardy–Weinberg law

1. Introduction

The use of prior information in inference is well established in the Bernoulli arena of statistical methodology. In some instances, prior information can be incorporated into classical models as well. For example, Dutta [1] considered an estimation of quadratic functions of the Bernoulli parameter in inverse sampling (IS) method that used the uniformly minimum variance unbiased (UMVU) estimators of $p^2 + \beta p$.

In this paper, we firstly also consider the Bernoulli population which has the parameter p and probability law as

$$f(x, p) = p^x (1 - p)^{1-x} \quad \text{for } x = 0, 1. \tag{1.1}$$

Let $x_1, x_2, x_3, \ldots$, be sequence of observations on random variable x. Moreover, we adopt the following two stopping rules to collect the samples from the population.

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Stopping rule I: Fixed size sampling (FSS) method

Continue taking up to n observations (a pre-assigned number), say $x_1, x_2, \ldots, x_n$. The sampling procedure is called as FSS method.

The probability of realization of a specific sample $x_1, x_2, \ldots, x_n$, such that $y = \sum_{i=1}^{n} x_i$, is given by

$$P(y,n) = \binom{n}{y} p^y (1-p)^{n-y} \quad \text{for } y = 0, 1, 2, \ldots, n. \tag{1.2}$$

Stopping rule II: Inverse sampling (IS) method

Continue taking observations in a sequence until a predetermined number c of observations fall in to a given class $x_i = 1$. The sampling procedure is called an inverse sampling (IS) method.

The probability of realization of a specific sample, $(x_1, x_2, x_3, \ldots, x_{c+z})$ is given by

$$P(z,c) = \binom{c+z-1}{z} p^c (1-p)^z \quad \text{for } z = 0, 1, 2, \ldots, \tag{1.3}$$

where z is the number of $x_i = 0$ in the sample.

Here y and z are two complete sufficient statistics for the family of distributions (1.2) and (1.3), respectively. Therefore, Dutta [1] suggest the UMVU estimator of $p^2 + \beta p$ by using IS method and FSS method. In addition, consistency is a desirable property of an estimator; and, in all cases of practical interest, maximum likelihood estimators (MLEs) are consistent. Moreover, MLEs have also the invariance property. Hence, in this paper, we consider the problem of MLE of a quadratic function of p, say $p^2 + \beta p$ by using IS method and FSS method. Moreover, we also compare the two sampling methods, that used the minimum mean squared error (MSE) of the precision of the MLEs, as the criterion for comparison. It is shown that the IS method is a competitor to the FSS method. The MLE of $p^2 + \beta p$ and their MSE can also be obtained in the Section 2, for the two sampling methods. In the Section 3, we also give a computational comparison of MSE for $p^2 + \beta p$ to assess the performance of two sampling methods by using numerical method.

The problem of estimation of quadratic functions of arises in genetics. Consider a single gene of two alleles, say A and a, where 'A' denotes an allele which is clinically dominating in nature (or which has recessive feature) and 'a' denotes an allele which is clinically recessive in nature (or which has recessive feature). In the absence of forces that change the gene frequencies, the relative frequencies of each gene allele tend to remain constant from generation to generation. This is known as Hardy–Weinberg law. According to which the probabilities of the occurrence of genotypes AA, Aa and aa are p^2, $2p(1-p)$ and $(1-p)^2$, respectively (see [2]).

2. The MLE of $p^2 + \beta p$ and their mean squared error in the two different sampling methods

The quadratic function p^2, $p(1-p)$ and $(1-p)^2 - 1$ can be written in a general from as $p^2 + \beta p$, with $\beta = 0, -1, -2$, respectively. Now, we introduce the MLE of the quadratic function $p^2 + \beta p$ under the two sampling methods. Let $g_{\mathrm{F}}(y)$ and $g_{\mathrm{I}}(z)$ denoted the MLE of $p^2 + \beta p$ (for different values of β) by using the FSS method and IS method, respectively.

(1) The FSS method

The MLE of $p^2 + \beta p$ in this method is given by

$$g_{\mathrm{F}}(y) = \left(\frac{y}{n}\right)^2 + \beta\left(\frac{y}{n}\right). \tag{2.1}$$

Furthermore, the MSE of $g_{\mathrm{F}}(y)$ is

$$\begin{aligned} \mathrm{MSE}_{\mathrm{F}} &= V_{\mathrm{F}}(g) + [Eg_{\mathrm{F}} - (p^2 + \beta p)]^2 \\ &= \frac{p^4}{n^3}[(4n-3)(2-n)] + \frac{p^3}{n}\left[\left(4 - \frac{18}{n} + \frac{12}{n^2}\right) + 4\beta\left(\frac{1}{n} - 1\right)\right] \\ &\quad + \frac{p^2}{n}\left[\frac{7(n-1)}{n^2} + \beta(4-\beta) - \frac{6\beta}{n}\right] + \frac{p}{n}\left(\beta + \frac{1}{n}\right)^2. \end{aligned} \tag{2.2}$$