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Solvability of p-Laplace equations subject to three-point boundary value problems

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Abstract

In this paper, the existences of solutions for p-Laplace equations subject to three-point boundary value conditions at resonance and non-resonance are studied by using degree theory and some known results are improved.
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Keywords: p-Laplace equations; Three-point boundary value; Resonance; Degree theory

1. Introduction and main results

The turbulent flow in a porous medium is a fundamental mechanics problem. For studying this type problem, Leibenson [1] introduced the following model

$$u_t = \frac{\partial}{\partial x}\left(\frac{\partial(u^m)}{\partial x}\left|\frac{\partial(u^m)}{\partial x}\right|^{p-1}\right), \tag{1.1}$$

where $m \geqslant 2$, $\frac{1}{2} \leqslant p \leqslant 1$. Generally, when $m > 1$, Eq. (1.1) is called porous medium equation [2]; when $0 < m < 1$, called diffused equation; when $m = 1$, called heat equation, which often appears in non-Newtonian liquid [3]. For the study of Eq. (1.1), ones reduced Eq. (1.1) into the following p-Laplace equation

$$(\phi_p(u'))' = f(t, u, u'), \quad t \in (0,1), \tag{1.2}$$

where $\phi_p(s) = |s|^{p-2}s$. Obviously, when $p = 2$, Eq. (1.2) becomes to the general second order differential equation.

In recent years, many important results relative to Eq. (1.2) with certain boundary conditions have been obtained (see [4–8,10], and references therein). Carcía-huidobro et al. [8] discussed the boundary value problems for p-Laplace Eq. (1.2) with boundary conditions

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$$u'(0) = 0, \quad u(1) = u(\eta), \quad \eta \in (0,1) \tag{1.3}$$

using the main assumptions as following:

(A_1) there are non-negative functions $d_1(t)$, $d_2(t)$, and $r(t) \in L^1[0,1]$ such that

$$|f(t,u,v)| \leqslant d_1(t)|u|^{p-1} + d_2(t)|v|^{p-1} + r(t), \quad \text{for a.e. } t \in [0,1], u, v \in \mathbb{R};$$

(A_2) there exists $u_0 > 0$, such that for all $|u| > u_0$, $t \in [0,1]$ and $v \in \mathbb{R}$, it holds that

$$|f(t,u,v)| \geqslant \Lambda|u|^{p-1} - A|v|^{p-1} - B,$$

where $\Lambda > 0$, and A, $B \geqslant 0$;

(A_3) there is $R > 0$ such that for all $|u| > R$, either

$$uf(t,u,0) > 0, \quad \text{a.e. } t \in [0,1]$$

or

$$uf(t,u,0) < 0, \quad \text{a.e. } t \in [0,1]$$

as well as other conditions.

In this paper, we discuss the solvability of the p-Laplace Eq. (1.2) at resonance and non-resonance, and obtain the following main results.

Theorem 2.1. *Assume that $f : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ is continuous and has the decomposition*

$$f(t,u,v) = g(t,u,v) + h(t,u,v)$$

such that

(H_1) *there exist $r_1 < 0$, $r_2 > 0$, such that*

$$f(t,r_1,0) \leqslant 0, \quad f(t,r_2,0) \geqslant 0, \quad \textit{for } t \in [0,1];$$

(H_2) *there is $R > 0$ such that*

$$vg(t,u,v) \leqslant 0, \quad \textit{for } (t,u) \in [0,1] \times [r_1,r_2], \ |v| > R;$$

(H_3) *there are non-negative functions $a(t), b(t) \in C([0,1], \mathbb{R}^+)$, such that for all $(t,u,v) \in [0,1] \times [r_1,r_2] \times \mathbb{R}$*

$$|h(t,u,v)| \leqslant a(t)|v|^m + b(t),$$

where $m \leqslant p$.

Then there exists a solution of BVP (1.2), (1.3).
And we also get the following theorem:

Theorem 3.1. *Let $f : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ is continuous. Assume that*

(H_4) *there exist $r_3 < 0$, $r_4 > 0$, such that*

$$f(t,r_3,0) < 0, \quad f(t,r_4,0) > 0, \quad \textit{for } t \in [0,1];$$

(H_5) *there are non-negative functions $c(t), d(t) \in C([0,1], \mathbb{R}^+)$, such that for all $(t,u,v) \in [0,1] \times [r_1,r_2] \times \mathbb{R}$*

$$|f(t,u,v)| \leqslant c(t)|v|^m + d(t),$$

where $m \leqslant p$.

Then there exists a solution of Eq. (1.2) with boundary conditions

$$u(0) = 0, \quad u(1) = u(\eta), \quad \eta \in (0,1). \tag{1.4}$$

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