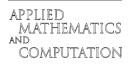


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## Solvability of *p*-Laplace equations subject to three-point boundary value problems

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## Abstract

In this paper, the existences of solutions for *p*-Laplace equations subject to three-point boundary value conditions at resonance and non-resonance are studied by using degree theory and some known results are improved. © 2005 Elsevier Inc. All rights reserved.

Keywords: p-Laplace equations; Three-point boundary value; Resonance; Degree theory

## 1. Introduction and main results

The turbulent flow in a porous medium is a fundamental mechanics problem. For studying this type problem, Leibenson [1] introduced the following model

$$u_t = \frac{\partial}{\partial x} \left( \frac{\partial(u^m)}{\partial x} \left| \frac{\partial(u^m)}{\partial x} \right|^{p-1} \right), \tag{1.1}$$

where  $m \ge 2$ ,  $\frac{1}{2} \le p \le 1$ . Generally, when  $m \ge 1$ , Eq. (1.1) is called porous medium equation [2]; when  $0 \le m \le 1$ , called diffusion equation; when m = 1, called heat equation, which often appears in non-Newtonian liquid [3]. For the study of Eq. (1.1), ones reduced Eq. (1.1) into the following *p*-Laplace equation

$$(\phi_p(u'))' = f(t, u, u'), \quad t \in (0, 1), \tag{1.2}$$

where  $\phi_p(s) = |s|^{p-2}s$ . Obviously, when p = 2, Eq. (1.2) becomes to the general second order differential equation.

In recent years, many important results relative to Eq. (1.2) with certain boundary conditions have been obtained (see [4–8,10], and references therein). Carcía-huidobro et al. [8] discussed the boundary value problems for *p*-Laplace Eq. (1.2) with boundary conditions

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$$u'(0) = 0, \quad u(1) = u(\eta), \quad \eta \in (0, 1)$$
(1.3)

using the main assumptions as following:

(A<sub>1</sub>) there are non-negative functions  $d_1(t)$ ,  $d_2(t)$ , and  $r(t) \in L^1[0,1]$  such that

$$|f(t, u, v)| \leq d_1(t)|u|^{p-1} + d_2(t)|v|^{p-1} + r(t), \text{ for a.e. } t \in [0, 1], u, v \in \mathbb{R};$$

(A<sub>2</sub>) there exists  $u_0 \ge 0$ , such that for all  $|u| \ge u_0$ ,  $t \in [0, 1]$  and  $v \in \mathbb{R}$ , it holds that

 $|f(t,u,v)| \ge \Lambda |u|^{p-1} - A|v|^{p-1} - B,$ 

where  $A \ge 0$ , and  $A, B \ge 0$ ; (A<sub>3</sub>) there is  $R \ge 0$  such that for all  $|u| \ge R$ , either

> uf(t, u, 0) > 0, a.e.  $t \in [0, 1]$ or uf(t, u, 0) < 0, a.e.  $t \in [0, 1]$

as well as other conditions.

In this paper, we discuss the solvability of the p-Laplace Eq. (1.2) at resonance and non-resonance, and obtain the following main results.

**Theorem 2.1.** Assume that  $f: [0,1] \times \mathbb{R}^2 \to \mathbb{R}$  is continuous and has the decomposition

$$f(t, u, v) = g(t, u, v) + h(t, u, v)$$

such that

(H<sub>1</sub>) there exist 
$$r_1 < 0$$
,  $r_2 > 0$ , such that  
 $f(t, r_1, 0) \leq 0$ ,  $f(t, r_2, 0) \geq 0$ , for  $t \in [0, 1]$ ;

(H<sub>2</sub>) there is  $R \ge 0$  such that

 $vg(t, u, v) \leq 0$ , for  $(t, u) \in [0, 1] \times [r_1, r_2]$ , |v| > R;

(H<sub>3</sub>) there are non-negative functions  $a(t), b(t) \in C([0, 1], \mathbb{R}^+)$ , such that for all  $(t, u, v) \in [0, 1] \times [r_1, r_2] \times \mathbb{R}$  $|h(t, u, v)| \leq a(t)|v|^m + b(t)$ , where  $m \leq p$ .

Then there exists a solution of BVP (1.2), (1.3). And we also get the following theorem:

**Theorem 3.1.** Let  $f : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$  is continuous. Assume that

 $(H_4)$  there exist  $r_3 < 0$ ,  $r_4 > 0$ , such that

 $f(t, r_3, 0) < 0$ ,  $f(t, r_4, 0) > 0$ , for  $t \in [0, 1]$ ;

(H<sub>5</sub>) there are non-negative functions  $c(t), d(t) \in C([0,1], \mathbb{R}^+)$ , such that for all  $(t, u, v) \in [0,1] \times [r_1, r_2] \times \mathbb{R}^+$ 

$$|f(t, u, v)| \leq c(t)|v|^m + d(t),$$
  
where  $m \leq p$ .

Then there exists a solution of Eq. (1.2) with boundary conditions

$$u(0) = 0, \quad u(1) = u(\eta), \quad \eta \in (0, 1).$$
 (1.4)

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