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Applied Mathematics and Computation 177 (2006) 63–78
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Impulsive effects on stability of Cohen–Grossberg neural networks with variable delays

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Abstract

In this paper, a model of impulsive Cohen–Grossberg neural networks is first formulated. By establishing some impulsive differential inequalities, we investigate impulsive effects on the stability of Cohen–Grossberg neural networks with variable delays and obtain some sufficient conditions ensuring global exponential stability of the impulsive delay system. Our criteria not only show that the stability still remains under certain impulsive perturbations for some continuous stable neural networks, but also present an approach to stabilize the unstable neural networks by utilizing impulsive effects. The results extend and improve some recent works for impulsive neural networks as well as non-impulsive neural networks. Some examples and their simulations are given for illustration of the theoretical results.

Keywords: Cohen–Grossberge neural networks; Impulsive effects; Delays; Global exponential stability; Stabilization; Impulsive differential inequalities

1. Introduction

Cohen–Grossberg neural network model was initially proposed by Cohen and Grossberg [1] in 1983 and soon has attracted considerable attention in theoretical research and engineering applications. Especially, delay neural network model has recently been deeply investigated since time delays are ubiquitous both in biological and artificial neural networks (see, e.g., [2–9]). On the other hand, besides delay effects, impulsive effects are also likely to exist in the neural network system [10–15]. For instance, in implementation of electronic networks, the state of the networks is subject to instantaneous perturbations and experiences abrupt change at certain instants, which may be caused by switching phenomenon, frequency change or other sudden noise, that is, does exhibit impulsive effects [16–19]. Even in biological neural networks, impulsive effects may be unavoidable. According to Arbib [26] and Haykin [27], when a stimuli from the body or the external environment is received by receptors, the electrical impulses will be conveyed to the neural net and impulsive effects arise naturally in the net. Consequently, Cohen–Grossberg neural network model with delays and impulsive effects

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.10.032

may be more accurate to describe the evolutionary process of the system. From the view of mathematical model, impulsive neural network model belongs to new category of dynamical systems, which is neither purely continuous-time nor purely discrete-time ones. Such a model displays a combination of characteristics of both the continuous-time and discrete-time systems and has complex dynamical behaviors. Therefore, it is necessary to further investigate Cohen–Grossberg neural network model with delays and impulsive effects.

In this paper, we consider a model of Cohen–Grossberg neural network system with variable delays and impulsive effects as follows:

$$\begin{cases} \dot{y}_i(t) = -a_i(y_i(t))\left[b_i(y_i(t)) - \sum\limits_{j=1}^{n} w_{ij}g_j(y_j(t-r_{ij}(t))) + J_i\right], & t \neq t_k,\ t \geqslant 0, \\ \Delta y_i = y_i(t_k) - y_i(t_k^-) = I_{ik}\big(y(t_k^-)\big), & i = 1,2,\ldots,n,\ k \in N \triangleq \{1,2,\ldots,\}, \end{cases} \tag{1}$$

where $y(t) = (y_1(t),\ldots,y_n(t))^{\mathrm{T}}$ is the neuron state, $\dot{y}_i$ denotes the derivative, w_{ij} represents weight of the neuron interconnection, J_i is the constant input from outside of the networks, a_i represents an amplification function, b_i is an appropriately behaved function, g_j denotes the activation function, $r_{ij}(t)$ is the transmission delay function, t_k is called impulsive moment, $y_i(t_k^-)$ and $y_i(t_k^+)$ denote the left-hand and right-hand limit at t_k, respectively, I_{ik} shows impulsive perturbation of the ith neuron at time t_k. We always assume $y_i(t_k) = y_i(t_k^+)$, $k \in N$. If $I_{ik}(y) = 0$ for all $y \in R^n$, $i = 1,2,\ldots,n$, $k \in N$, then the model (1) becomes continuous Cohen–Grossberg neural networks

$$\dot{y}_i(t) = -a_i(y_i(t))\left[b_i(y_i(t)) - \sum_{j=1}^{n} w_{ij}g_j(y_j(t-r_{ij}(t))) + J_i\right], \quad t \geqslant 0, \tag{2}$$

which contains some popular models such as Hopfield neural networks, bidirectional neural networks, Lotka–Volterra competition models and other recurrent neural network models.

It is well known that stability analysis of neural networks is a prerequisite for the practice design and applications. A number of criteria on the stability has been derived for continuous Cohen–Grossberg neural network as well as continuous Hopfield-type neural networks in the literature [1–9,20–25]. Since Eq. (1) may be treated as the impulsively perturbed system of Eq. (2), it is interesting to further investigate how the impulsive perturbations affect the stability property of (2). More precisely, it is useful to know

(i) whether the perturbed system (1) remains stable when the continuous system (2) is stable;
(ii) whether the impulsive system (1) becomes stable when the system (2) is unstable.

These two cases actually involve robustness of stability and impulsive stabilization of neural networks, respectively. Recently, the robustness of stability has been investigated by Guan et al. [10,11], Akca et al. [12], Gampasymy [13], Li et al. [14,15] for impulsive Hopfield-type neural networks. Some interesting results on the stability of impulsive neural networks have been obtained. However, these results may require stricter conditions on the continuous portion or impulsive portion in the impulsive networks. Furthermore, almost all the results assume in advance that the corresponding continuous system is stable (even exponentially stable). In fact, even if the corresponding continuous system is unstable, the system (1) can be stabilized by utilizing impulsive effects, but there are few results developed in this direction.

In this paper, we first develop two impulsive differential inequalities. Based on the inequalities, we obtain less restrictive sufficient condition of global exponential stability for impulsive system (1). Moreover, we initially investigate impulsive stabilization of neural networks. Finally, examples and comparisons are given to demonstrate our main results.

2. Preliminaries

In this section, we shall introduce some basic definitions and assumptions and present two lemmas.

$PC[J,R^n] \triangleq \{u(t)\colon J \to R^n | u(t)$ is continuous at $t \neq t_k$, $u(t_k^+) = u(t_k)$ and $u(t_k^-)$ exists for t, $t_k \in J$, $k \in N\}$, where $J \subset R$ is an interval.