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Reflection of generalized thermoelastic waves from a solid half-space under hydrostatic initial stress

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Abstract

The basic governing equations for isotropic and homogeneous generalized thermoelastic media under hydorstatic initial stress are formulated in context of Lord–Shulman theory. These governing equations are solved analytically to obtain the dimensional velocities in x–y plane. It is shown that there exists three plane waves, namely, thermal wave, P wave and SV wave. Reflection from insulated stress-free surface is studied to obtain the reflection coefficients of the reflected waves for the incidence of thermal wave. The numerical computations are carried out for a particular model. The Effect of hydrostatic initial stress is observed on these reflected waves.

Keywords: Hydrostatic; Initial stress; Reflection; Relaxation time; Thermoelasticity

1. Introduction

Duhamel [1] and Neumann [2] introduced the theory of uncoupled thermoelasticity. There are two shortcomings of this theory. First, the fact that the mechanical state of the elastic body has no effect on the temperature is not in accordance with true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation for the temperature, which is not physically admissible.

Biot [3] developed the coupled theory of thermoelasticity which deals with the first defect of uncoupled theory, but shares the second defect of uncoupled theory. Lord and Shulman [4] introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. A more rigorous theory of thermoelasticity by introducing two relaxation times has been formulated by Green and Lindsay [5]. These theories allows finite speed of propagation of waves. Chandrasekharaiah [6] referred to this wavelike thermal disturbance as "second sound". A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak [7].

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The wave propagation in thermoelastic media is of much importance in various fields such as earthquake engineering, soil dynamics, aeronautics, astronautics, nuclear reactors, high energy particle accelerator, etc. Various authors have worked on wave propagation in isotropic thermoelasticity. For example, Sinha and Sinha [8] and Sinha and Elsibai [9] studied problems on reflection of thermoelastic waves at a solid half-space in context of Lord and Shulman [4] and Green and Lindsay [5] theories. Sharma et al. [10] reviewed the problem of Sinha and Sinha [8] for various linear theories of thermoelasticity. Sinha and Elsibai [11] investigated the reflection of thermoelastic waves at the interface of two semi-infinite media in welded contact. Singh [12] and Abd-alla et al. [13] studied some problems on reflection of the generalized magneto-thermo-viscoelastic plane waves from stress-free surface. The study of wave propagation in an isotropic generalized thermoelastic solid with additional parameters provide information about existence of new or modified waves. This information may be useful for experimental seismologists in correcting earthquake estimation.

Initial stresses are developed in the medium due to many reasons, resulting from difference of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations, etc. The Earth is supposed to be under high initial stresses. It is therefore of great interest to study the effect of these stresses on the propagation of stress waves. During the last five decades considerable attention has been directed toward this phenomena. It was the achievement of Biot [14] to show the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. He has obtained the velocities of longitudinal and transverse waves along the coordinate axis only.

It is not possible to review the whole work done on propagation of waves in an unbounded medium under initial stresses. The work related to the present paper is reviewed only. The study of reflection and refraction phenomena of plane waves in an unbounded medium under initial stresses is due to Chattopadhyay et al. [15], Sidhu and Singh [16], Dey et al. [17].

Recently, Montanaro [18] investigated the isotropic linear thermoelasticity with hydrostatic initial stress. The present paper is organized as follows: First, the governing equations given by Montanaro [18] are modified in light of Lord and Shulman [4] theory for linear, isotropic and homogeneous case. Secondly, the governing equations are solved analytically for two-dimensional motion in x–y plane to obtain the expressions for dimensional velocities of three plane waves. Lastly, the expressions involving reflection coefficients are calculated both theoretically as well as numerically. The numerical results are shown graphically to show the effect of initial stress upon the reflection coefficients.

2. Governing equations

Following Lord and Shulman [4] and Montanaro [18], the constitutive relations and field equations for homogeneous, isotropic thermoelastic solid with hydrostatic initial stress and in absence of incremental body forces and heat sources are

$$\sigma_{ij} = -p(\delta_{ij} + \omega_{ij}) + \bar{\lambda} e_{pp}\delta_{ij} + 2\bar{\mu} e_{ij} - \frac{\alpha}{\kappa_T}\Theta\delta_{ij}, \tag{1}$$

$$h_i = K\frac{\partial \Theta}{\partial x_i}, \tag{2}$$

$$U = \frac{\alpha T_0}{\rho_0 \kappa_T} e_{pp} + c_e\Theta, \tag{3}$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{4}$$

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right), \tag{5}$$

$$\rho_0\frac{\partial^2 u_i}{\partial t^2} = \left(\bar{\mu} - \frac{p}{2}\right)\frac{\partial^2 u_i}{\partial x_p \partial x_p} + \left(\bar{\lambda} + \bar{\mu} + \frac{p}{2}\right)\frac{\partial^2 u_p}{\partial x_i \partial x_p} - \frac{\alpha}{\kappa_T}\frac{\partial \Theta}{\partial x_i}, \tag{6}$$

$$\rho_0 c_e\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial \Theta}{\partial t} + \frac{\alpha T_0}{\kappa_T}\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial^2 u_p}{\partial t \partial x_p} = K\frac{\partial^2 \Theta}{\partial x_p \partial x_p}, \tag{7}$$