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# Reconstruction of permittivity profile from boundary capacitance data

Weifu Fang

*Department of Mathematics, West Virginia University, Morgantown, WV 26506, USA*

**Abstract**

We present numerical methods of reconstructing permittivity profile of material flow from data collected in electrical capacitance tomography (ECT). Based on a formula for the sensitivity maps from a partial differential equation model, we propose a unified framework of iterative algorithms using updated sensitivity maps to properly incorporate the nonlinear nature of the problem and a regularization mechanism to address the ill-posedness. For two- and three-phase flows, the total variation (TV) regularization method is shown to be effective in reconstructing the permittivity profiles accurately in a few iterations, without any a priori knowledge of the unknown permittivity values.


*Keywords:* Electrical capacitance tomography; Sensitivity map; Image reconstruction; Inverse problem; Regularization; Total variation regularization

## 1. Introduction

In electrical capacitance tomography (ECT), the main objective is to reconstruct the permittivity profile of material flow inside the sensor from capacitance data collected by the electrode patches on the exterior boundary of the sensor. This technique has found many applications in various industrial processes, and there is a large volume of literature on this problem (see e.g. review articles [1,5,10,15,16]).

In a typical setup of an ECT sensor, several thin rectangular metal electrodes are placed around the exterior wall of an insulating pipe or vessel, within which certain material flow is steadily passing through. The electrode patches are usually sufficiently long along the length of the vessel that the cross-sectional model is assumed appropriate. The most common type of vessels is circular in cross-section, although vessels with square cross-sections have also been reported in some laboratory settings. The number of electrodes varies from applications, usually ranging from 8 to 32. A schematic of the cross-section of a circular sensor with 12 electrodes is depicted in Fig. 1. Certain patterns of voltage excitation are applied to the electrode patches to induce charges to be measured on the electrodes, which are then converted into capacitance data. In the single excitation mode, the applied voltage pattern is 1 (normalized) on one electrode (source) and 0 on the

*E-mail address:* wfang@math.wvu.edu

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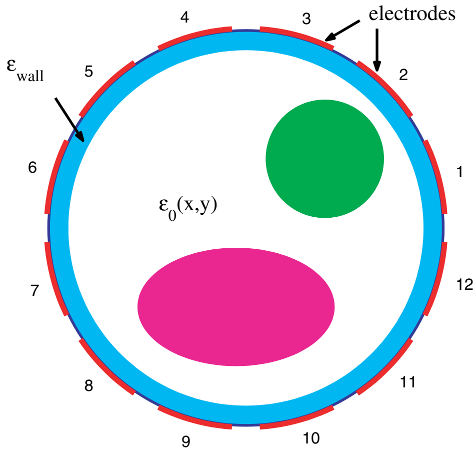


Fig. 1. Cross-section of a 12-electrode circular ECT sensor.

remaining electrodes (detectors), and the induced charges on the detector electrodes are measured. The designated source electrode is alternating until each electrode in the sensor system has served once as a source electrode, thus completing the collection of all mutual capacitance measurements between any pair of electrodes. The entire sensor is usually enclosed by a screen to shield off other stray electromagnetic fields; Although possible, we do not include the screen shield in our model here for the sake of simplicity in presentation. Therefore, the capacitance measurements are directly related to the permittivity distribution of the material flowing through the vessel. The objective of ECT is to extract the material permittivity profile information from the capacitance data, by designing effective and efficient algorithms that are capable of reconstructing cross-sectional permittivity profiles from collected capacitance measurements. In the forward problem, we are to simulate the capacitance data from a given permittivity distribution, while in the inverse problem, we wish to reconstruct the permittivity from a given set of capacitance measurements. In applications, the permittivity cross-sectional profile is often displayed as a two-dimensional image for visualization, hence a process of solving the inverse problem is commonly referred to as an image reconstruction algorithm. The recent review article [15] gives an overview of the current state of image reconstruction algorithms in this area.

From the view point of mathematical modeling, this problem is very similar to electrical impedance tomography (EIT), where it is the interior conductivity profile that is to be recovered from boundary voltage responses to certain current excitation patterns on the patches. There have been many studies for the EIT problem, in both engineering and mathematical fields (see e.g. [2] and references therein). Both tomography systems can be modeled by similar boundary value problems for elliptic partial differential equations, although the application areas and thus the demands from applications are different.

In this paper we present numerical methods of reconstructing the permittivity distribution from the capacitance data. In particular, we design algorithms that are appropriate to ECT, and more specifically, to permittivity reconstruction of multi-phase material flows. Following the partial differential equation model formulated in [8], we derive formulas for the sensitivity maps. In this formulation, the vessel wall is included as part of the model, and hence the effect of the wall material and thickness can be examined. The formulas for the sensitivity maps lead to familiar forms of the linear model that have been widely used in the literature. This linear model is not accurate enough for most algorithms to capture the permittivity satisfactorily, and the nonlinear nature of the maps need to be incorporated whenever possible. We propose a unified framework of iterative algorithms using updated sensitivity maps in each iteration, with a proper regularization form. The updates on the sensitivity maps incorporate naturally the nonlinearity of the model in the algorithms, and the regularization is necessary for solving the ill-posed problem. As for the regularization, we choose the total variation (TV) regularization method over the common Tikhonov regularization for multi-phase flows, since it is known that Tikhonov regularization is more suitable for recovering smooth profiles, while TV regularization is more effective for functions with "blocky" images such as the permittivity profile of multi-phase flows. It will also be seen that it takes only a few iterations for the algorithms to