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A Taylor collocation method for the approximate solution of general linear Fredholm–Volterra integro-difference equations with mixed argument

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Abstract

In this paper, a Taylor collocation method is developed to find an approximate solution of high-order linear Fredholm–Volterra integro-difference equations with variable coefficients under the mixed conditions. The solution is obtained in terms of Taylor polynomial near any point. Also, examples are presented which illustrate the partinent features of the method and the results are discussed.

Keywords: Taylor polynomials; Fredholm–Volterra difference and integro-difference equations; Collocation points

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1. Introduction

Taylor and Chebyshev collocation methods for the approximate solutions of differential, integral and integro-differential equations have been presented in many papers [1–4]. Also, a Taylor expansion approach to solve high-order linear difference equations has been presented by Gülsu and Sezer [5].

In this paper, the methods in the mentioned studies are developed and applied to the high order non-homogenous linear Fredholm–Volterra difference equation with variable coefficients and mixed argument

$$\sum_{k=0}^{m} P_k(x)y(x+k) + \sum_{j=1}^{J} Q_j(x)y(x-j) = g(x) + \int_a^b K(x,t)y(t)\,\mathrm{d}t + \int_a^x K^*(x,t)y(t)\,\mathrm{d}t, \tag{1}$$

which is the extended case of the equations given in [6, p. 174] and [7, p. 176], with the mixed conditions

$$\sum_{r=0}^{R} c_{lr}y(c_r) = \mu_l, \quad l = 0,1,\ldots,m+J-1, \;\; a \leqslant c_r \leqslant b \tag{2}$$

and the solution is expressed in the form

$$y(x) = \sum_{n=0}^{N} a_n(x-c)^n, \tag{3}$$

so that the Taylor coefficients to be determined are

$$a_n = \frac{y^{(n)}(c)}{n!}, \quad n = 0,1,\ldots,N. \tag{4}$$

Here $P_k(x)$, $Q_j(x)$, $g(x)$, $K(x,t)$ and $K^*(x,t)$ are functions that are defined on interval $\min(-j,k) = a \leqslant x,\ t \leqslant b = \max(-j,k)$ and c_{lr} and μ_l are appropriate constants. In addition, to compute the coefficients a_n, we use the collocation points

$$x_i = a + \frac{b-a}{N}i, \quad i = 0,1,\ldots,N, \tag{5}$$

where $a \leqslant x_i \leqslant b$ and $a = x_0 < x_1 < \cdots < x_n = b$.

2. Fundamental matrix equations

Let us first write Eq. (1) in the form

$$D_1(x) + D_2(x) = g(x) + F(x) + V(x), \tag{6}$$