Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 175 (2006) 730–747
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

A finite element method for equal width equation

Bülent Saka

Mathematics Department, Osmangazi University, 26480 Eskişehir, Turkey

Abstract

A numerical solution of the equal width (EW) equation is obtained using space-splitting technique and quadratic B-spline Galerkin finite element method. Solitary wave motion, interaction of two solitary waves, wave undulation and wave generation are studied using the proposed method. Comparisons are made with analytical solutions and with some spline finite element method calculations at selected times. Accuracy and efficiency are discussed by computing the numerical conserved laws and L_2, L_∞ error norms.

Keywords: EW equation; Finite elements; Galerkin; Splines; Solitary waves

1. Introduction

The EW equation

$$U_t + \varepsilon U U_x - \mu U_{xxt} = 0, \tag{1}$$

E-mail address: bsaka@ogu.edu.tr

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.07.034

where the subscripts t and x denote differentiation and ε, μ are positive parameters, was introduced by Morrison et al. [3] as a model equation to describe the nonlinear dispersive waves. This equation is an alternative form of nonlinear dispersive waves to the well known regularized long wave equation and Korteweg–de Vries equation. These equations have solitary wave solutions which are wave packets or pulses. These waves propagate in nonlinear media by keeping wave forms and velocity even after interaction occurs. Few analytical solutions of the EW equation are known. So numerical methods are useful tool for study for the EW equation. Some numerical works of the equation have been presented: Galerkin's method using cubic B-splines as trial and test functions is set up to solve the EW equation by Gardner and Gardner [4]. Petrov–Galerkin method accompanied with quadratic B-splines is given in the paper [6]. Zaki obtained the numerical solution of the EW equation by using the method of least-squares [7]. A spectral method of the equation is presented by Garcia-Archilla [5]. A solution based on a collocation method incorporated cubic B-splines is investigated by Dağ and Saka [10]. Esen [11] applied a lumped Galerkin method based on quadratic B-spline finite elements has been used for solving the EW equation. The migration, interaction and generation of the solitary waves and undular bore development are studied in these papers.

In the present paper we set up finite element solution using quadratic B-splines as the element and the weight function for the space splitted EW equation. Using the Crank–Nicolson scheme for nodal parameters and finite difference scheme for the time integration, the resulting system of the ordinary differential equation is discretised to lead to nonlinear system of algebraic equations. This system is solved with the Gauss elimination method. Nonlinear part of the system is handled by using an inner iteration, which will be defined later.

2. Quadratic B-spline Galerkin method

To solve the EW Eq. (1) numerically, we replaced the setting $V(x,t) = -U_x(x,t)$ in Eq. (1). Hence we obtained a system of differential equations which have first order time and space derivatives

$$\begin{aligned} &U_t - \varepsilon UV + \mu V_{xt} = 0, \\ &V + U_x = 0. \end{aligned} \tag{2}$$

Boundary and initial conditions can be written as

$$\begin{aligned} &U(a,t) = \beta_1, \quad U(b,t) = \beta_2, \quad V(a,t) = V(b,t) = 0, \qquad t \in (0,T], \\ &U(x,0) = f(x), \quad V(x,0) = -f'(x), \qquad a \leqslant x \leqslant b. \end{aligned} \tag{3}$$