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On computing of arbitrary positive integer powers for one type of even order symmetric circulant matrices—I

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Abstract

In this paper we derive the general expression of the l-th power ($l \in N$) for one type of symmetric circulant matrices of order $n = 2p$ ($p \in N$, $p \geqslant 2$).

Keywords: Circulant matrices; Eigenvalues; Eigenvectors; Jordan's form; Chebyshev polynomials

1. Introduction

Solving some difference, differential equations and delay differential equations we meet the necessity to compute the arbitrary positive integer powers of square matrix [1–3]. In this paper we derive the general expression of the l-th power ($l \in N$) for one type of symmetric circulant matrices [4] of even order.

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2. Derivation of general expression

Consider the n-th order ($n = 2p$, $p \in N$, $p \geqslant 2$) symmetric circulant matrix B of the following type:

$$B = \begin{pmatrix} 0 & 1 & & & & 1 \\ 1 & 0 & 1 & & & \\ & 1 & 0 & 1 & & \\ & & & \ddots & & \\ & & & 1 & 0 & 1 \\ 1 & & & & 1 & 0 \end{pmatrix}. \tag{1}$$

The l-th power ($l \in N$) of this matrix we will find using the expression $B^l = TJ^lT^{-1}$ [5], where J is the Jordan's form of B, T is the transforming matrix. Matrices J and T can be found provided eigenvalues and eigenvectors of the matrix B are known. The eigenvalues of B are defined by the characteristic equation

$$|B - \lambda E| = 0; \tag{2}$$

here E is the identity matrix of the n-th order.

Let us denote

$$D_n(\alpha) = \begin{vmatrix} \alpha & 1 & & & & 1 \\ 1 & \alpha & 1 & & & \\ & 1 & \alpha & 1 & & \\ & & & \ddots & & \\ & & & 1 & \alpha & 1 \\ 1 & & & & 1 & \alpha \end{vmatrix}, \quad \Delta_n(\alpha) = \begin{vmatrix} \alpha & 1 & & & \\ 1 & \alpha & 1 & & 0 \\ & 1 & \alpha & 1 & \\ & & & \ddots & \\ & 0 & & 1 & \alpha & 1 \\ & & & & 1 & \alpha \end{vmatrix}; \tag{3}$$

here $\alpha \in R$. Then

$$|B - \lambda E| = D_n(-\lambda). \tag{4}$$

From (3) follows

$$D_n = \alpha\Delta_{n-1} - 2\Delta_{n-2} - 2(-1)^n$$

and

$$\Delta_n = \alpha\Delta_{n-1} - \Delta_{n-2} \ (\Delta_2 = \alpha^2 - 1, \Delta_1 = \alpha, \Delta_0 = 1); \tag{5}$$

here $D_n = D_n(\alpha)$, $\Delta_n = \Delta_n(\alpha)$. Solving difference Eq. (5) we obtain $\Delta_n(\alpha) = U_n(\frac{\alpha}{2})$,

$$D_n(\alpha) = U_n\left(\frac{\alpha}{2}\right) - U_{n-2}\left(\frac{\alpha}{2}\right) - 2(-1)^n; \tag{6}$$