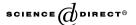
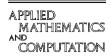


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# On computing of arbitrary positive integer powers for one type of even order symmetric circulant matrices—I

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#### Abstract

In this paper we derive the general expression of the *l*-th power  $(l \in N)$  for one type of symmetric circulant matrices of order n = 2p  $(p \in N, p \ge 2)$ . © 2005 Elsevier Inc. All rights reserved.

Keywords: Circulant matrices; Eigenvalues; Eigenvectors; Jordan's form; Chebyshev polynomials

#### 1. Introduction

Solving some difference, differential equations and delay differential equations we meet the necessity to compute the arbitrary positive integer powers of square matrix [1–3]. In this paper we derive the general expression of the l-th power ( $l \in N$ ) for one type of symmetric circulant matrices [4] of even order.

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#### 2. Derivation of general expression

Consider the *n*-th order  $(n = 2p, p \in N, p \ge 2)$  symmetric circulant matrix *B* of the following type:

$$B = \begin{pmatrix} 0 & 1 & & & & 1 \\ 1 & 0 & 1 & & & & \\ & 1 & 0 & 1 & & & \\ & & & \ddots & & & \\ & & & 1 & 0 & 1 \\ 1 & & & & 1 & 0 \end{pmatrix}. \tag{1}$$

The *l*-th power  $(l \in N)$  of this matrix we will find using the expression  $B^l = TJ^l\mathsf{T}^{-1}$  [5], where *J* is the Jordan's form of *B*, *T* is the transforming matrix. Matrices *J* and *T* can be found provided eigenvalues and eigenvectors of the matrix *B* are known. The eigenvalues of *B* are defined by the characteristic equation

$$|B - \lambda E| = 0; (2)$$

here E is the identity matrix of the n-th order.

Let us denote

$$D_{n}(\alpha) = \begin{vmatrix} \alpha & 1 & & & & 1 \\ 1 & \alpha & 1 & & & \\ & 1 & \alpha & 1 & & \\ & & \ddots & & \\ & & & 1 & \alpha & 1 \\ 1 & & & & 1 & \alpha \end{vmatrix}, \quad \Delta_{n}(\alpha) = \begin{vmatrix} \alpha & 1 & & & \\ 1 & \alpha & 1 & & 0 \\ & 1 & \alpha & 1 & \\ & & \ddots & & \\ & 0 & & 1 & \alpha & 1 \\ & & & & 1 & \alpha \end{vmatrix}; \quad (3)$$

here  $\alpha \in R$ . Then

$$|B - \lambda E| = D_n(-\lambda). \tag{4}$$

From (3) follows

$$D_n = \alpha \Delta_{n-1} - 2\Delta_{n-2} - 2(-1)^n$$

and

$$\Delta_n = \alpha \Delta_{n-1} - \Delta_{n-2} \ (\Delta_2 = \alpha^2 - 1, \Delta_1 = \alpha, \Delta_0 = 1); \tag{5}$$

here  $D_n = D_n(\alpha)$ ,  $\Delta_n = \Delta_n(\alpha)$ . Solving difference Eq. (5) we obtain  $\Delta_n(\alpha) = U_n(\frac{\alpha}{2})$ ,

$$D_n(\alpha) = U_n(\frac{\alpha}{2}) - U_{n-2}(\frac{\alpha}{2}) - 2(-1)^n;$$
 (6)

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