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Computer aided solving the high-order transition probability matrix of the finite Markov chain ☆

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Abstract

In this paper, by using theories of stochastic process and computer algebra, a reliable algorithm for computing the high-order transition probability matrix of finite Markov chain is established, a new mechanical procedure `markovproc` is established, too. The procedure `markovproc` give not only the exact expression of the Markov chains, but also the limiting probability. Some examples are presented to illustrate the implementation of the algorithm.

Keywords: Markov chain; Transition probability matrix; Algorithm; Mechanization; Maple

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1. Introduction

In recent years there has been a growing interest in the Markov chain. Markov chains are stochastic process that can be parameterized by empirically estimating transition probabilities between discrete states in the observed systems. Roughly speaking, for the Markov process, the probability of the given condition in the given moment may be deduced from information about the proceeding conditions. A Markov chain represents a system of elements moving from one state to another over time. Markov dependence has been used to model data in such diverse fields as work force planning, production, finance, accounting, education, marketing and health services, animals distribution, etc. [1–6].

But it is a difficult point in the field of computing the high-order transition probability matrix of finite Markov chain because of huge size of calculations. It is well known that the rapid development of computer science and computer algebra system has a profound effect on the concept and the methods of mathematical researches [7–15]. The objective of this paper is to establish a promising algorithm that can be easily computed the transition probability matrix in `Maple` with mechanization.

2. Basic methods

Let us first recall the basic principles of the method for computing the transition probability matrix of the Markov chain. Since these results are the key to the manipulation for our problems, we give them in details.

Let $X(t)$ be a stochastic process, possessing discrete states spaces $S = \{1, 2, \ldots, s\}$. In general, for a given sequence of time points $t_1 < t_2 < \cdots < t_{n-1} < t_n$, the conditional probabilities should be:

$$\begin{aligned} &P_r\{X(t_n) = i_n | X(t_1) = i_1, \ldots, X(t_{n-1}) = i_{n-1}\} \\ &\quad = P_r\{X(t_n) = i_n | X(t_{n-1}) = i_{n-1}\}. \end{aligned} \tag{2.1}$$

The conditional probabilities $P_r\{X(u) = j | X(v) = i\} = P_{ij}(v, u)$ are called transition probabilities of order $r = u - v$ from state i to state j for all indices $0 \leqslant v < u$, with $1 \leqslant i$ and $j \leqslant s$. They are denoted as the transition matrix P. For k states, the first-order transition matrix P has a size of $s \times s$ and takes the form:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1s} \\ p_{21} & p_{22} & \cdots & p_{2s} \\ \cdots & \cdots & \cdots & \cdots \\ p_{s1} & p_{s2} & \cdots & p_{ss} \end{bmatrix}. \tag{2.2}$$