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Computational dynamics of three-dimensional closed-chains of rigid bodies

Hazem Ali Attia

Department of Mathematics, College of Science, Al-Qasseem University, P.O. Box 237, Buraidah 81999, Saudi Arabia

Abstract

In this paper, the dynamic analysis of three-dimensional closed-chains of rigid bodies using computational approach is presented. The method uses the concepts of linear and angular momentums to generate the rigid body equations of motion in terms of the Cartesian coordinates of a dynamically equivalent constrained system of particles, without introducing any rotational coordinates and the corresponding rotational transformation matrix. Closed-chain system is transformed to open-chain by cutting suitable kinematical joints and introducing cut-joint constraints. For the resulting open-chain system, the equations of motion are generated recursively along the serial chains. An example is chosen to demonstrate the generality and simplicity of the developed formulation.

Keywords: Dynamic analysis; Equations of motion; Recursive formulation; Spatial motion; Mechanisms; Machine; Open-chains; Closed-chains

E-mail address: ah1113@yahoo.com

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1. Introduction

There are different formulations for the dynamic analysis of spatial mechanisms which vary in the system of coordinates used and in the way they introduce kinematical constraint equations [1–5]. Each formulation has its own advantages and disadvantages depending on the application and the needs. Some formulations are developed using a two-step transformation which leads to a simple and reduced system of equations. One method [6,7] uses initially the absolute coordinate formulation where the location of each rigid body in the system is described in terms of a set of translational and rotational coordinates. Then, the equations of motion are transformed to a reduced set in terms of the relative joint variables. Another method uses initially the point coordinate formulation in which a dynamically equivalent constrained system of particles replaces the rigid bodies [8–11]. The global motion of the constrained system of particles together with the constraints imposed upon them represent both the translational and rotational motions of the rigid body. The external forces and couples acting on the body are distributed over the system of particles. Then, the equations of motion which are expressed in terms of the Cartesian coordinates of the particles are rederived in terms of the relative joint variables. The main disadvantage of this two-step transformation is the necessity to transform at every time step from the joint variables to the original system which is time consuming.

A recursive dynamical formulation for the dynamic analysis of planar mechanisms is presented by Attia [12]. The concepts of linear and angular momentum are used to write the rigid body dynamical equations without the need to distribute the external forces and couples over the particles. The method can be applied to recursively generate the equations of motion for open and/or closed-chain systems.

In this paper, the dynamic analysis of three-dimensional of closed-chain of rigid bodies is presented using computational approach. The method is based upon the idea of replacing the rigid body by its dynamically equivalent constrained system of particles discussed in [8–12] with essential modifications and improvements. The concepts of the linear and angular momentums are used to formulate the rigid body dynamical equations. However, they are expressed in terms of the rectangular Cartesian coordinates of the equivalent constrained system of particles. This groups the advantages of the automatic elimination of the unknown internal forces as in Newton–Euler formulation and results in a reduced system of differential-algebraic equations. Some useful geometrical relationships are used to obtain a reduced dynamically equivalent constrained system of particles.

For the closed-chain system, the system is transformed to open-chain system by cutting suitable kinematical joints and introducing the cut-joint kinematical constraints. For the resulting open-chain system, the equations of motion are