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Series approximations for the means of k-records

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Abstract

Applying two methods, wavelet and Taylor expansion, approximations for expectations of k-record values are derived. Since ordinary record values are contained in the k-records, approximations for their moments can be obtained as particular case. In the case of wavelet expansion, the lower and upper bounds for the mean of k-records are obtained. Numerical examples are presented in order to compare approximations and exact values for means with respect to underlying logistic distribution.

Keywords: Haar wavelet; Logistic distribution; Order statistics; Record values; Taylor expansion; Wavelet expansion

1. Introduction

Let $\{X_i, i \geqslant 1\}$ be a sequence of independent and identically distributed (iid) continuous random variables each distributed according to cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$. An

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observation X_j will be called an *upper record value* if its value exceeds all previous observations. Thus, X_j is an *upper record* if $X_j > X_i$ for every $i < j$. An analogous definition can be given for *lower record values*.

Record data arise in a wide variety of practical situations. Examples include industrial stress testing, meteorological analysis, hydrology, seismology, sporting and athletic events, and oil and mining surveys. Properties of record data have been studied extensively in the literature. First, Chandler [8] studied the stochastic behavior of random record values arising from the "classical record model", that is, the record model where the underlying sample from which records are observed comprises of iid observations from a continuous probability distribution. Since then, there are over 500 papers and many books published on record-breaking data. Interested readers may refer to the books of Arnold et al. [6] and Nevzorov [17], see also Ahmadi [1]. In the record value theory, while inverse sampling considerations have given valuable insights, their practical implementation is greatly hindered by the sparsity of records. In fact, the expected waiting time is infinite for every record after the first; but, this problem will be fixed by considering k-records instead (see Theorem 2.1 of [14]). Upper k-record process is defined in terms of the kth largest X yet seen. For a formal definition, we consider the definition in Arnold et al. [6, p. 43], in the continuous case, let $T_{1,k} = k$, $R_{1,k} = X_{1:k}$ and for $n \geqslant 2$, let

$$T_{n,k} = \min\{j : \; j > T_{n-1,k}, X_j > X_{T_{n-1,k}-k+1:T_{n-1,k}}\},$$

where $X_{i:m}$ denotes the ith order statistic in a sample of size m. (Theorem 2.1 [14], *For* $k \geqslant 2$, $E(T_{m,k})$ *is finite for all* m).

The sequence of *upper k-records* are then defined by $R_{n,k} = X_{T_{n,k}-k+1:T_{n,k}}$ for $n \geqslant 1$, Arnold et al. [6] call them *Type 2 k-record* sequence. For $k = 1$, note that the usual records are recovered. An analogous definition can be given for *lower k-records* as well. These sequence of k-records were introduced by Dziubdziela and Kopocinski [12] and it have found acceptance in the literature. The theory of kth records is still developing. In reliability analysis, the life length of the r-out-of-n system is the $(n - r + 1)$th order statistics in a sample of size n. Therefore, the nth k-record value can be regarded as just the life length of a k-out-of-$T_{n,k}$ system.

There are many applications that require knowledge of moments of record statistics. For example, coefficients of the best linear unbiased estimators of the location and scale parameters of a distribution require these moments. They are also used in a similar way in prediction problems. Several authors have discussed the subject of moments bounds and approximations of order statistics. In the context of record statistics, Nagaraja [16] presented analytic formulae for the sharp bounds of record statistics, based on application of the Schwarz inequality (see also [7]). By the same approach, Ahmadi and Arghami [2] obtained the universal bounds for the moments of difference of record values. Raqab [19] evaluated bounds on expectations of record statistics based on