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Computing queue length distributions in $MAP/G/1/N$ queue under single and multiple vacation

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Abstract

This paper studies a single server queue with finite waiting room in which the server takes vacation(s) whenever the system becomes empty and we consider both single and multiple vacation(s). Whereas the input process is a Markovian Arrival Process (MAP), the service and vacation times are arbitrarily distributed. The distributions of number of customers in the queue at service completion, vacation termination, departure, arbitrary and pre-arrival epochs have been obtained. Computational procedure has been given when the service- and vacation-time distributions are of phase type (PH-distribution).

Keywords: Finite buffer; Markovian arrival process; Queue; Single server; Single and multiple vacation

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1. Introduction

Queueing systems with vacations have been studied extensively in the past several years and applied to many areas including computer-communications and manufacturing systems. An excellent survey on this topic can be found in Doshi [5]. In this connection see also books by Takagi [22,24] and references therein. Most of the analysis in the past have been carried out assuming Poisson input and considering infinite queue capacity. However, in many applications arrival process need not be Poisson e.g. in modern communication systems and Asynchronous Transfer Mode (ATM) networks, arrivals to a statistical multiplexer are correlated. The Markovian Arrival Process (MAP) is a representative of correlated arrivals and includes many familiar input processes such as Markov modulated Poisson process (MMPP), PH-type renewal process, Poisson process etc. It was first introduced by Lucantoni et al. [14]. In recent years there has been a great interest in analyzing queueing systems with vacations and MAP as input process: $MAP/G/1$ queue see e.g., Lucantoni et al. [14], Kasahara et al. [9] and Lee et al. [11]. Few authors, Matendo [16,17], Ferrandiz [6], Schellhaas [21], etc., have studied queueing systems with vacations assuming input as batch Markovian arrival process (BMAP).

The finite buffer queues are more realistic and have wide applications in many areas mentioned above. Blondia [1] analyzed $MAP/G/1/N$ queue with multiple vacations and obtained the queue length distributions at departures-, arbitrary- and pre-arrival-epochs. He discussed two types of vacations models (i) exhaustive service discipline and (ii) limited service discipline. Further, he obtained the Laplace–Stieltjes transform (LST) of virtual and actual waiting time distributions. A more general $MAP/G/1/N$ queue with single (multiple) vacation(s) along with setup and close-down time have been discussed by Niu and Takahashi [20] using supplementary variable technique whereby they obtained queue length distributions at arbitrary epoch and LST of virtual and actual waiting time distributions.

In this paper we consider $MAP/G/1/N$ queue with single and multiple vacation(s) and carry out its detailed analysis including computational aspects. The analytic analysis of this queue is carried out and the distributions of the number of customers in the queue at service completion, vacation termination and departure epochs have been obtained using the imbedded Markov chain technique. The supplementary variable (with remaining service time of a customer in service, and remaining vacation time of the server as supplementary variables) method is used to develop the relations between the queue length distributions when the server is busy/vacation at arbitrary and departure epochs. Though these relations can also be obtained using other methods such as renewal theory, see, for example, Blondia [1]. The advantage of using supplementary variable method over other methods is that one can obtain several other results as a byproduct by using simple algebraic manipulation of transform