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Semi-analytical solution of a constrained fourth-order integro-differential equation of steady flow–structure interaction in a model collapsible tube

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Abstract

We provide accurate, non-perturbative, semi-analytical solutions to a constrained fourth-order integro-differential equation derived by Djorjevic and Vukobratovic [On a steady, viscous flow in two-dimensional channels, Acta Mech. 163 (2003) 189–205] in the study of a steady incompressible flow through a flexible 2D tube. Our numerical experiments motivate the introduction of a global parameter that monitors the deformation of the tube as a function of material properties and flow characteristics. Physically, this parameter is proportional to the work done by the longitudinal tension on the wall. An example is provided on how this global parameter can be used to monitor coronary lesion progression. In particular, this parameter is used to define an index of criticality of such lesions.

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1. Introduction

The majority of physiological flows occurs in deformable channels. In order to monitor and possibly treat diseases such as sleep apnea (snooring), aneurysm and coronary artery occlusion (stenosis), an understanding of the interaction between collapsible channels and biological fluid flow is therefore essential. Shah and Humphreh [1] studied the flow–structure interaction for spherical intercranial aneurysms. Their numerical experiment points to the dynamical stability of such lesions. In the case of coronary artery disease, in vivo experiments suggest that regions of high shear stress on the innermost arterial wall are considered vulnerable to plaque rupture while regions of low shear stress are condusive to plaque growth [2]. This paper describes a semi-analytical technique for modeling fluid–structure interaction in a collapsible tube.

There are several approaches to the modeling of flows in flexible tube. Some of these approaches are succintly described below. The reader is referred to the recent paper by Grotberg and Jensen [3] for a comprehensive review.

The equations governing the fluid–structure coupling are the Navier–Stokes equations for the flow and Cauchy's first law of motion for the tube wall. These equations are supplemented with the appropriate constitutive relations and initial/boundary conditions. They are highly nonlinear and difficult to solve analytically even for low-dimensional (1D and 2D) situations. Besides, the problem is a free-boundary one. Indeed the deformable tube geometry and the flow characteristics must be simultaneously determined.

It should be noted that for one-dimensional flows the wall–flow interaction is typically modeled as follows. The unknowns are the cross-sectional area and the average cross-sectional velocity which are obtained by solving the conservation equations of mass and momentum. However, the conservation of momentum in this case deviates from the Navier–Stokes equation in that the viscous dissipation is specified in an ad-hoc fashion. In addition, a relationship between the average cross-sectional pressure and the cross-sectional area is assumed. This formulation is the so-called "tube's law".

The analytical study of flows in flexible channels is challenging. Neverthless, in some situations, it is possible to drastically simplify the governing equations in such a way that analytical or semi-analytical techniques become applicable. For 2D, steady, incompressible, low Reynolds number and lubrication type flow, Djordjevic and Vukobratovic [4] derived a single constrained fourth-order integro-differential equation describing the tube configuration. Specifically, for a system of the type illustrated in Fig. 1 they obtained the equation