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Approximation of solutions to non-local history-valued retarded differential equations

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Abstract

In the present work we consider a retarded differential equation and prove the existence, uniqueness and convergence of approximate solutions. Also we consider the Faedo–Galerkin approximations of the solution and the convergence.

Keywords: Retarded differential equation; Analytic semigroup; Approximate solution; Faedo–Galerkin approximation and convergence

1. Introduction

In this paper we consider the following retarded differential equation with a non-local history condition in a separable Hilbert space H:

$$\begin{aligned} &u'(t) + Au(t) = f(t, u(t), u_t), \quad t \in (0, T], \\ &g(u_0) = \phi, \ \text{on} \ [-\tau, 0], \end{aligned} \tag{1.1}$$

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where the nonlinear map f is defined from $[0,T]\times D(A^{\alpha})\times C_0^{\alpha}$ into H, C_0^{α} being the space of all continuous functions from $[-\tau,0]$ into $D(A^{\alpha})$ endowed with the norm

$$\|\psi\|_{0,\alpha} := \sup_{-\tau\leqslant\theta\leqslant 0}\|A^{\alpha}\psi(\theta)\|,\quad \psi\in C_0^{\alpha},$$

map g is defined from C_0 into C_0, $\phi\in C_0$ and $-A$ is the infinitesimal generator of a strongly continuous semigroup.

For $t\in[0,T]$, we shall use the notation $C_t := C([-\tau,t];H)$ for the Banach space of all continuous functions from $[-\tau,t]$ into H endowed with the supremum norm

$$\|\psi\|_t := \sup_{-\tau\leqslant\eta\leqslant t}\|\psi(\eta)\|,\quad \psi\in C_t.$$

Similarly for $t=0$ we shall use the notation $C_0 := C([-\tau,0];H)$ for the Banach space of all continuous functions from $[-\tau,0]$ into H endowed with the supremum norm

$$\|\psi\|_0 := \sup_{-\tau\leqslant\eta\leqslant 0}\|\psi(\eta)\|,\quad \psi\in C_0.$$

For the work on existence, uniqueness and regularity of solutions of functional differential equation under different conditions, we refer to Blasio and Sinestrari [1] and Jeong et al. [2] and reference cited in these papers.

Also for the further works on existence, uniqueness and stability of various types of solutions of differential and functional differential equations, we refer to Bahuguna [3], Balachandran and Chandrasekaran [4], Lin and Liu [5], Alaoui [6] and references cited in these papers.

The related results for the approximation of solutions may be found in Bahuguna et al. [7] and Bahuguna and Shukla [8].

Initial studies concerning existence, uniqueness and finite-time blow-up of solutions for the following equation:

$$\begin{aligned} u'(t)+Au(t) &= g(u(t)),\quad t\geqslant 0,\\ u(0) &= \phi, \end{aligned}\tag{1.2}$$

have been considered by Segal [9], Murakami [10] and Heinz et al. [11]. Bazley [12,13] has considered the following semilinear wave equation:

$$\begin{aligned} &u''(t)+Au(t) = g(u(t)),\quad t\geqslant 0,\\ &u(0)=\phi,\quad u'(0)=\psi \end{aligned}\tag{1.3}$$

and has established the uniform convergence of approximations of solutions to (1.3) using the existence results of Heinz et al. [11]. Goethel [14] has proved the convergence of approximations of solutions to (1.2) but assumed g to be defined on the whole of H. Based on the ideas of Bazley [12,13], Miletta [15] has proved the convergence of approximations to solutions of (1.2). In the