Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 174 (2006) 566–577
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Computing the least-square solutions for centrohermitian matrix problems ☆

Zhongyun Liu [*,1], Zhaolu Tian, Yanxiang Tan

School of Mathematics and Computing Science, Changsha University of Science and Technology Changsha, Hunan 410076, PR China

Abstract

In this paper, we propose several algorithms for computing the solutions of the following three problems:

Problem I: Given $X, B \in \mathbf{C}^{n \times m}$ $(n > m)$, find a centrohermitian matrix $A \in \mathbf{C}^{n \times n}$ such that $\|AX - B\| = \min$.

Problem II: Given $X, B \in \mathbf{C}^{n \times m}$ $(n > m)$, find a centrohermitian matrix $A \in \mathbf{C}^{n \times n}$ such that $AX = B$.

Problem III: Let $\mathscr{S}$ be the solution set of Problem I or Problem II. Given $\widetilde{A} \in \mathbf{C}^{n \times n}$, find $A^* \in \mathscr{S}$ such that $\|\widetilde{A} - A^*\| = \inf_{A \in \mathscr{S}} \|\widetilde{A} - A\|$, where $\| \cdot \|$ is the Frobenius norm.

We show that our algorithms ensure significant savings in computational costs, as compared to the case of an arbitrary matrix A.

Keywords: Centrohermitian matrix; Optimal approximation; Least-squares solution

☆ Partially supported by the National Natural Science Foundation of China, No. 10271021.
* Corresponding author.
E-mail address: liuzhongyun@263.net (Z. Liu).
[1] Supported by the Excellent Youth Foundation of Educational Committee of Hunan Province (No. 04B006).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.04.104

1. Introduction

Consider the solutions of the following general form of the Problem I (Problem IV): *Given* $X, B \in \mathbf{C}^{n\times m}$ $(n > m)$, *find a matrix* $A \in \mathbf{C}^{n\times n}$ *such that* $\|AX - B\| = \min$. There are three standard ways to solve it. They are the normal equations, the QR decomposition, and the singular value decomposition (SVD), respectively. Each way has its advantage and disadvantage, which depends on the rank and condition number of the coefficient matrix X, and the relative importance of speed and reliability to the user (see [2] for details). If X has full column rank m, one can use the normal equations or the QR decomposition to solve the Problem IV. If X is not of full rank, then the SVD is an adequate choice.

To be specific, we take example for SVD to develop our algorithms. The corresponding algorithms by the normal equations or the QR decomposition can be developed by a similar way.

Assume that the SVD of the matrix X is

$$X = \widehat{U}\begin{pmatrix} \widehat{\Sigma} & 0 \\ 0 & 0 \end{pmatrix}\widehat{V}^H = \widehat{U_1}\widehat{\Sigma}\widehat{V_1}^H, \tag{1}$$

where $\widehat{U} = (\widehat{U_1}, \widehat{U_2})$, $\widehat{V} = (\widehat{V_1}, \widehat{V_2})$ are, respectively, $n\times n$ and $m\times m$ unitary matrices with $\widehat{U}_1 \in \mathbf{C}^{n\times\hat{r}}$, $\widehat{V}_1 \in \mathbf{C}^{m\times\hat{r}}$, $\hat{r} = \text{rank}(X)$, $\widehat{\Sigma} = \text{diag}(\hat{\sigma}_1, \ldots, \hat{\sigma}_{\hat{r}})$, $\hat{\sigma}_i > 0$, $1 \leqslant i \leqslant \hat{r}$. Then the general solution of Problem IV can be written as

$$A = B\widehat{V_1}\widehat{\Sigma}^{-1}\widehat{U_1}^H + C\widehat{U_2}^H, \tag{2}$$

where C is any of all $n \times (n - \hat{r})$ complex matrices.

The conventional algorithm based on the SVD for computing the problem IV can be outlined as follows:

Algorithm 1. *This algorithm for computing a solution of Problem IV.*

Input X, B and C.
Step 1. *Compute the SVD of the matrix X according to (1).*
Step 2. *Compute A according to (2).*

The number of operations to compute a solution of the Problem IV consists of two parts. One is the computational costs of the SVD in (1). It takes $nm^2 - \frac{1}{3}m^3$ complex multiplications and about the same number of complex additions when $n \gg m$, and about $2nm^2 - \frac{2}{3}m^3$ complex multiplications and about the same number of complex additions for smaller n. Another is the operations of solving Eq. (2). By the conventional algorithm for computing matrix–matrix multiplication, it performs $(m + 1)n\hat{r} + n^3$ complex multiplications and