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A new algorithm for the discrete fuzzy shortest path problem in a network

Tzung-Nan Chuang [a,*], Jung-Yuan Kung [b]

[a] *Department of Merchant Marine, National Taiwan Ocean University, Keelung 20224, Taiwan*
[b] *Department of Information Management, Chinese Naval Academy, Kaohsiung, 81342, Taiwan*

Abstract

In a network, the arc lengths may represent time or cost. In practical situations, it is reasonable to assume that each arc length is a discrete fuzzy set. We called it the discrete fuzzy shortest path problem. There were several methods reported to solve this kind of problem in the open literature. In these methods, they can obtain either the fuzzy shortest length or the shortest path. In this paper, we propose a new algorithm that can obtain both of them. The discrete fuzzy shortest length method is proposed to find the fuzzy shortest length, and the fuzzy similarity measure is utilized to get the shortest path. An illustrative example is given to demonstrate our proposed algorithm.

Keywords: The shortest path problem; Discrete fuzzy sets; Fuzzy similarity

1. Introduction

In the past decades, much attention has been paid to the shortest path problem in a network since it is important to a great deal of applications such as

* Corresponding author.
E-mail addresses: chu@mail.ntou.edu.tw (T.-N. Chuang), jykung@mail.cna.edu.tw (J.-Y. Kung).

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routing, communication, and transportation [1]. In a network, the shortest length is the minimum of all path lengths. In the crisp world, it is apparent to obtain the shortest length. For example, there are two path lengths in a network, $l_1 = 4$ and $l_2 = 5$. The shortest length is the minimum of l_1 and l_2. It is trivial to obtain the shortest length is 4. However, in real world, the arc length in the path of network may represent cost or time and it can be considered to be a discrete fuzzy set. If the arc lengths are fuzzy, it is not so obvious to obtain the shortest length. For example, in a network, there are two path lengths that are discrete fuzzy sets L_1 and L_2. Assume $L_1 = \{1.0/5, 0.6/6\}$ and $L_2 = \{1.0/4, 0.7/7\}$, that is, L_1 has lengths 5 and 6 with a membership grade of 1.0 and 0.6, respectively; L_2 has lengths 4 and 7 with a membership grade of 1.0 and 0.7, separately. The shortest length is the minimum of L_1 and L_2. The length 4 in L_2 is a possible shortest length since it is smaller than both of the possible lengths in L_1. The length 7 in L_2 is, however, not a possible shortest one since it is larger than both of the possible lengths in L_1. On the other hand, both the lengths 5 and 6 in L_1 are possible shortest length since they are smaller than the possible length 7 in L_2. The lengths, 4, 5 and 6 are then possible shortest length while 7 is not. As mentioned above, L_1 has two possible shortest lengths 5 and 6; L_2 has one possible shortest length 4. Thus, neither L_1 nor L_2 can represent the final set of possible shortest length. In this paper, we propose a discrete fuzzy shortest length method to find the fuzzy shortest length in a network.

Lots of approaches were developed to deal with the fuzzy shortest path problem [3–9]. Dubois and Prade [3] presented a method based on Floyd's algorithm and Ford's algorithm [1,2] to treat the fuzzy shortest path problem. Their method can obtain the shortest path length whereas the corresponding shortest path in the network perhaps does not exist. Later, Klein [7] proposed an improved algorithm that was based on dynamical programming recursion. Klein's algorithm can get not only the shortest path length but also the corresponding shortest path in the network; nevertheless, the assumption that the possible arc lengths are 1 through a fixed integer seems to be impractical. In this paper, to overcome the drawback mentioned above, we develop a method to determine the fuzzy shortest length from source node to the destination node in the network. Then, by means of fuzzy similarity measure to evaluate similarity degree between fuzzy lengths, we decide a practical shortest path whose length is closest to the yielded fuzzy shortest length.

The rest of this paper is organized as follows. In Section 2, some related fuzzy set operations and fuzzy similarity measure are reviewed. In Section 3, a method to find the discrete fuzzy shortest length is presented. In Section 4, through combining the previous method with fuzzy similarity measure, a new algorithm is developed to get the shortest length as well as the shortest path. An illustrative example is also included to demonstrate our proposed algorithm. Finally, some conclusions are drawn.