Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 173 (2006) 683–693
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Approximate solution of general high-order linear nonhomogeneous difference equations by means of Taylor collocation method

Mustafa Gülsu *, Mehmet Sezer, Zekeriya Güney

Department of Mathematics, Faculty of Science, Mugla University, Mugla, Turkey

Abstract

In this paper, a Taylor collocation method is developed to find an approximate solution of general high-order linear nonhomogenous difference equations with variable coefficients under the mixed conditions. The solution is obtained in terms of Taylor polynomials about any point. Also, examples are presented which illustrate the pertinent features of the method.

Keywords: Taylor polynomials; Difference equations; Collocation methods

1. Introduction

Taylor and Chebyshev collocation methods for the approximate solutions of differential, integral and integrodifferential equations have been presented in many papers [1–4].

* Corresponding author.
E-mail address: mgulsu@mu.edu.tr (M. Gülsu).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.04.048

On the other hand, a Taylor expansions approach to solve high-order linear difference equations has been presented by Gülsu and Sezer [5]. In this paper, these methods are developed and applied to the mth-order linear nonhomogenous difference equation with variable coefficients, which is given in [6, p. 174] and [7, p. 176],

$$\sum_{k=0}^{m} P_k(x)y(x+k) = f(x), \quad k \geqslant 0, \ k \in N^+ \tag{1}$$

with the mixed conditions

$$\sum_{j=0}^{p} a_{ij}y(c_j) = \mu_i, \quad a \leqslant c_j, \ x \leqslant b; \ i = 0, 1, \ldots, m-1 \tag{2}$$

and the solution is expressed as the Taylor polynomial

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n, \quad a \leqslant x, \ c \leqslant b \tag{3}$$

so that $y^{(n)}(c)$, $n = 0, 1, \ldots, N$ are the coefficients to be determined. Here $P_k(x)$ and $f(x)$ are functions defined on $a \leqslant x \leqslant b$; the real coefficients a_{ij}, c_j and μ_i are appropriate constants.

2. Fundamental matrix relations

Let us first consider the desired solution $y(x)$ of Eq. (1) defined by a truncated Taylor series (3). Then the solutions $y(x)$ can be expressed in the matrix form

$$[y(x)] = \mathbf{X}\mathbf{M}_0\mathbf{A}, \tag{4}$$

where

$$\mathbf{X} = [\,1 \quad (x-c) \quad (x-c)^2 \quad \ldots \quad (x-c)^N\,],$$
$$\mathbf{A} = [\,y^{(0)}(c) \quad y^{(1)}(c) \quad \ldots \quad y^{(N)}(c)\,]^{\mathrm{T}}$$

and

$$M_0 = \begin{bmatrix} \frac{1}{0!} & 0 & 0 & . & . & . & 0 \\ 0 & \frac{1}{1!} & 0 & . & . & . & 0 \\ 0 & 0 & \frac{1}{2!} & . & . & . & 0 \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ 0 & 0 & 0 & . & . & . & \frac{1}{N!} \end{bmatrix}.$$