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Linear objective function optimization with fuzzy relation equation constraints regarding max–av composition

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Abstract

In this paper, an optimization model with a linear objective function subject to a system of the fuzzy relation equations with max–av composition is presented. The solution set of such a fuzzy relation equations is a non-convex set. In this paper, firstly we discuss the feasible solution set with two schemes and, secondly, study relationship between maximum and minimum points, and also, the feasible points as well. Furthermore, an algorithm and few concrete examples are presented in order to optimize linear objective function.

Keywords: Linear objective function optimization; Fuzzy relation equations

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1. Introduction

Let $A = (a_{ij})_{m\times n}$, $0 \leqslant a_{ij} \leqslant 1$, be a fuzzy matrix and $b = (b_1, b_2 \ldots, b_n)$, $0 \leqslant b_j \leqslant 1$, be an n-dimensional vector, then the fuzzy relation equations is introduced as follows:

$$x \underset{\text{av}}{\circ} A = b,$$

where $\underset{\text{av}}{\circ}$ is max–av composition [16]. In this paper, we intend to find the solution vector $x = (x_1, x_2, \ldots, x_m)$, $0 \leqslant x_i \leqslant 1$, such that

$$\max_{i=1,\ldots,m} (x_i + a_{ij}) = 2b_j, \quad j = 1, \ldots, n.$$

The fuzzy relation equations topic is one of the interesting subjects and is investigated by many researchers [1–4,7–15]. In this paper we will give one kind of such problems.

Let $c = (c_1, c_2, \ldots, c_m)$ is an m-dimensional vector where c_j is cost coefficient associated with variable x_j. We want to study the problem below:

$$\begin{aligned} \min \quad & z = \sum_{i=1}^{m} c_i x_i, \\ & x \underset{\text{av}}{\circ} A = b, \\ & 0 \leqslant x_i \leqslant 1. \end{aligned}$$

We realize the quite different characteristic of the linear optimization with fuzzy relation equations when compare it with regular linear programming problems [5]. According to [3,8], the non-empty solution set of the fuzzy relation equations is in general a non-convex set, and can be expressed in terms of the maximum solution and the finite number of minimum solutions regarding max–min composition. Because the feasible solution set is non-convex, traditional linear programming methods, such as simplex and interior-point algorithms, become useless. These facts are true for fuzzy relation equations regarding max–av composition, as well.

In this paper, at first, in Sections 2 and 3, we study the feasible solution set with two different schemes that call them straight schemes. Either scheme accomplishes the other one and second scheme is designed in the base of the first.

Then, in Section 4 we present an algorithm to find the real optimal point without attempting to find all the potential minimum points. we present some numerical examples to illustrate these schemes and the related theorems and also apply the given algorithm for that kind of the problems whose objective functions is linear and cost coefficients are non-negative, primarily. In considering the general problems, at first, we decompose the main problem into two sub-problems. One of them with non-negative cost coefficients and other one