Available online at www.sciencedirect.com

SCIENCE DIRECT®

Applied Mathematics and Computation 176 (2006) 642–653

APPLIED MATHEMATICS AND COMPUTATION

www.elsevier.com/locate/amc

Least-squares νth-order polynomial estimation of signals from observations affected by non-independent uncertainty

S. Nakamori [a,*], R. Caballero-Águila [b], A. Hermoso-Carazo [c], J. Jiménez-López [b], J. Linares-Pérez [c]

[a] Department of Technology, Faculty of Education, Kagoshima University, 1-20-6, Kohrimoto, Kagoshima 890-0065, Japan
[b] Departamento de Estadística e I.O., Universidad de Jaén, Paraje Las Lagunillas, s/n, 23071 Jaén, Spain
[c] Departamento de Estadística e I.O., Universidad de Granada, Campus Fuentenueva, s/n, 18071 Granada, Spain

Abstract

The least-squares νth-order polynomial filtering and fixed-point smoothing problems of uncertainly observed discrete-time signals are considered, when the variables describing the uncertainty in the observations are non-independent. By defining suitable augmented signal and observation vectors, the polynomial estimation problem of the signal is reduced to the linear estimation problem of the augmented signal. The proposed estimators do not require the knowledge of the state-space model generating the signal, but only the probability that the signal exists in the observations, the $(2,2)$ element of the conditional probability matrices of the sequence describing the uncertainty and the moments (up to the 2νth ones) of the signal and the observation noise.

Keywords: Least-squares estimation; Uncertain observations; Polynomial estimation; Covariance information; Conditional probability

1. Introduction

The estimation problem in linear stochastic systems has been traditionally addressed under the assumption of gaussianity of the processes involved; in such case it is well known that the least-squares (LS) estimator of the signal is a linear function of the observations and can be recursively obtained by the Kalman filter. Nevertheless, in non-gaussian systems, the LS estimator is not easily obtainable in general, and this difficulty motivates the necessity of looking for suboptimal estimators which can be easily derived and which improve the widely used linear ones. In this context, the polynomial estimation problem has played an important role and has been studied by several authors; specifically, De Santis et al. [1] established a recursive algorithm for

* Corresponding author.
E-mail addresses: nakamori@edu.kagoshima-u.ac.jp (S. Nakamori), raguila@ujaen.es (R. Caballero-Águila), ahermoso@ugr.es (A. Hermoso-Carazo), jdomingo@ujaen.es (J. Jiménez-López), jlinares@ugr.es (J. Linares-Pérez).

0096-3003/$ - see front matter © 2005 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2005.10.009

the LS second-order polynomial filter in non-gaussian systems and Carravetta et al. [2] generalized these results considering the polynomial filtering problem with arbitrary degree. This polynomial estimation theory has been used to solve different signal processing problems, such as image processing [3], reconstruction and noise reduction of signals [4] or image restoration [5], among others.

Systems with uncertain observations constitute an interesting class of non-gaussian systems characterized by including in the observation equation a multiplicative noise modelled by a binary random sequence taking the values one or zero (that is, a sequence of Bernoulli random variables); the value one indicates that the corresponding observation contains the signal, whereas the value zero reflects the fact that the signal is absent and, consequently, the observation is only noise. The Bernoulli random variables modelling the uncertainty in the observations may be independent, but there are many practical situations (for instance, problems of fading or reflection of signals transmitted from the ionosphere, or signal transmission in multichannel systems) where such variables follow a certain dependence structure. Under the assumption that the uncertain in the observations is modelled by independent random variables, both the LS linear and polynomial estimation problems have been treated by several authors, as NaNacara and Yaz [6] or Caballero et al. [7], among others. On the other hand, Hadidi and Schwartz [8] were the first who studied the LS linear estimation problem when the variables modelling the uncertainty in the observations are not necessarily independent; they proved that, in general, the linear estimators do not admit a recursive structure similar to that of the Kalman filter and established a necessary and sufficient condition for such recursivity.

In all these papers the estimation problem is addressed assuming a full knowledge of the state-space model for the signal process. Nevertheless, as it is well known, this model can be unavailable in some practical situations, and the study of the estimation problem must be based on another kind of information, for example covariance information about the processes involved. When the Bernoulli variables describing the uncertainty in the observations are independent, the LS linear estimation problem using this kind of information has been considered, for example, in [9]; this study is generalized in [10] by deriving recursive algorithms for the LS second-order polynomial estimators, which improve on the linear ones. Assuming the same dependence structure on the Bernoulli variables as that considered by Hadidi and Schwartz [8], both estimation problems (linear and second-order polynomial) have been also addressed in [11,12] using only information about the covariance functions of the processes involved.

The aim of this paper is to generalize the results in [12] by considering the LS polynomial estimation problem of arbitrary degree (ν) from uncertain observations, when the variables modelling the uncertainty are non-independent. More specifically, the polynomial filtering and fixed-point smoothing problems are addressed under the assumption that the variables describing the uncertainty in the observations satisfy the condition established by Hadidi and Schwartz [8]. For this purpose, the technique proposed by Carravetta et al. [2], consisting of augmenting the signal and observation vectors with their Kronecker powers, is used. This technique allows to reduce the polynomial estimation problem to the linear estimation problem of the augmented signal. So, a recursive algorithm for the linear filter and fixed-point smoother of this augmented signal is proposed. Besides the probability that the signal exists in the observations, the estimators only require the $(2,2)$ element of the conditional probability matrix of the sequence describing the uncertainty and the moments (up to the 2νth ones) of the signal and the observation additive noise. The effectiveness of the second-order and third-order polynomial estimators in contrast to the linear ones is shown by applying the algorithm to the estimation problem of a signal transmitted in a multichannel system.

2. Observation equation and problem statement

Let $z(k)$ and $y(k)$ be $n \times 1$ vectors describing the signal and its observation at time k, respectively. Suppose that the observation equation is given by

$$y(k) = U(k)z(k) + v(k),$$

where the signal $\{z(k);\ k \geqslant 0\}$ is a zero-mean stochastic process, the additive noise $\{v(k);\ k \geqslant 0\}$ is a zero-mean white sequence and the multiplicative noise $\{U(k);\ k \geqslant 0\}$ is a sequence of Bernoulli random variables with $P[U(k) = 1] = p(k) \neq 0$, satisfying that the conditional probability $P[U(k) = 1 | U(j) = 1] = P_{2,2}(k)$ is independent of j for $j < k$ (this condition guarantees that the linear estimators admit a recursive structure similar to