Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 172 (2006) 1257–1271
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

A substitutable two-product inventory system with joint-ordering policy and common demand

V.S.S. Yadavalli [a,*], C. De W Van Schoor [a], S. Udayabaskaran [b]

[a] *Department of Industrial and Systems Engineering, University of Pretoria, 0002 Pretoria, South Africa*
[b] *Department of Mathematics, Presidency College, University of Madras, Chennai 600 005, India*

Abstract

A substitutable two-product inventory system with joint-ordering policy is considered in this paper. Common demands occur according to a Poisson process. A demand is satisfied either with an item of product 1 with probability p_1 or with an item of product 2 with probability p_2 ($p_1 + p_2 = 1$). When one of the products is out of stock, the demand is satisfied with the other available product with probability 1. Analyzing the imbedded renewal process describing the system, expressions for the stationary distribution of the inventory level and the stationary rates of the replenishments, the re-orders placed, the lost demands, and the units replenished are obtained. A cost analysis is also provided. Numerical example illustrated the results obtained.

* Corresponding author.
E-mail addresses: sarma.yadavalli@up.ac.za (V.S.S. Yadavalli), chris.vanschoor@up.ac.za (C. De W Van Schoor), s_udayabaskaran@yahoo.com (S. Udayabaskaran).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.02.064

1. Introduction

In the study of multi-product inventory systems, the concept of common demand for some products arises. For example, when a desired customer arrives at a shop, which sells two brands of soft drinks, he/she may be satisfied by a soft drink of a particular brand with probability p_1, or by the other with probability p_2, $0 < p_i < 1$, $p_1 + p_2 = 1$. If any one of the products is out of stock, due to the desire, the customer will accept with probability 1 the other product which may be available in the shop. Also when the supplier is the same for several products under consideration, the dealer would prefer to have a simultaneous replenishment of all the products due to several reasons like cost considerations. Joint-ordering policies for periodic inventory systems have been studied by several researchers [1,2]. Parlar and Weng [3] developed optimal coordination policies for the supply and manufacturing departments. They considered a problem where the responsibility of the manufacturing department was to meet the random demand of a product with a short life cycle. The responsibility the supply department was to provide a sufficient amount of raw materials, so that the required production level could be achieved. Yadavalli and Joubert [4] studied a problem of joint coordination between manufacturing and supplying department encountered in a short life cycle multi-product environment. On the other hand, the study of continuous review multi-product inventory systems with common demand has not been considered so far in the literature. In this paper, an attempt is made to fill the gap by providing a study of a substitutable two-product inventory system with joint-ordering policy and common demand. The layout of the paper is as follows: In Section 2, the model assumptions and notation are provided. Certain auxiliary functions, which characterize the occurrence of various events pertaining to the model are derived in Section 3. Section 4 gives some of the measures of the performance of the system. A cost analysis is provided in Section 5. Section 6 deals with the numerical results, which highlights the behaviour of the system.

2. The model and assumptions

We consider a continuous review two-product inventory model with the following assumptions:

(i) The maximum inventory level of product i is S_i, $i = 1, 2$.
(ii) Demands occur according to a Poisson process with parameter λ.
(iii) When both products are available, a demand is satisfied either with product 1 with probability p_1, or with product 2 with probability p_2, $0 < p_i < 1$, $p_1 + p_2 = 1$. When one of the products is out of stock, the demand is