Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER Applied Mathematics and Computation 173 (2006) 19–32

APPLIED MATHEMATICS AND COMPUTATION

www.elsevier.com/locate/amc

A simple similarity-transformation-iterative scheme applied to Korteweg–de Vries equation

T. Öziş [a], S. Özer [b,*]

[a] *Department of Mathematics, Faculty of Science, Ege University, 35100 İzmir, Turkey*
[b] *Department of Mathematics, Faculty of Arts and Science, İnönü University, 44280 Malatya, Turkey*

Abstract

In this paper, we apply a simple iterative scheme and give an algorithm to solve the reduced Korteweg–de Vries equation by a scaled similarity-transformation. The obtained numerical solutions are valid for the whole solution domain. Results obtained by the method have been compared with the solution of Marchant and Smyth [T.R. Marchant, N.F. Smyth, Initial-boundary value problems for the KdV equations, IMA J. Appl. Math. 47 (1991) 247–264] and are found to be in good agreement with each other.

Keywords: Korteweg–de Vries equation; Similarity-transformation; Finite-difference

* Corresponding author.
E-mail addresses: ozis@sci.ege.edu.tr (T. Öziş), sozer@inonu.edu.tr (S. Özer).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.02.059

1. Introduction

In 1895 Korteweg and de Vries [2] derived an equation (the so-called KdV equation) which models one-dimensional shallow water waves with small but finite amplitudes. Recently, the KdV equation has been used to describe a number of important physical phenomena: for example, magnetohydrodynamic waves in a warm plasma, acoustic waves in an anharmonic crystal and ion-acoustic waves. Some remarkable papers, investigating various aspects of the above, can be found in Refs. [3–8].

The KdV equation has also been discussed for interaction between nonlinearity and dispersion, just as in Burgers' equation (see e.g., [9]), shows the features of the interaction between nonlinearity and dissipation. Moreover the KdV equation has been studied analytically by several authors. For example, existence and uniqueness of a solution for certain initial and boundary conditions were given by Kametaka [10], Sjöberg [11] and Menikoff [12]. The properties of the solution of the KdV equation were studied by Zabusky and Kruskal [13], Lax [14], Gardner et al. [15] and others. The numerical solutions of the KdV equation have also been investigated by many mathematicians and physicists. For example, Vliengenhart [16], Goda [17], Greig and Morris [18] gave finite-difference solutions and Alexander and Morris [19], Serna and Christie [20], Schoombie [21] and Gardner and Ali [22] obtained finite-element solutions of the KdV equation. More recently, Adomian [23], Wazwaz [24] and Kaya [25] gave solutions of the KdV equation by the decomposition method.

In this paper, we have given the solution of the KdV equation by applying a simple iterative scheme to the reduced Korteweg–de Vries equation by a scaled similarity-transformation.

2. General remarks and model problem

The KdV equation for long waves in shallow water can be given by

$$u_t + \sqrt{gh_0}\left[1 + \frac{3}{2}(u/h_0)\right]u_s + \frac{1}{6}\sqrt{gh_0}{h_0}^2 u_{sss} = 0, \tag{1}$$

where s denotes the coordinate along the horizontal line, t is the time and $u(s,t)$ is the local wave-height. The constants h_0 and g are undisturbed depth and acceleration of gravity respectively. Eq. (1) is valid for small values of a/h_0 and $(h_0/\lambda_0)^2$ where a and λ_0 denote the amplitude and wave length, respectively. Since the similarity reduction method is completely standard (see, for example, [26]), one of the derivation of Korteweg–de Vries equation can be obtained by introducing dimensionless variables

$$\bar{s} = s/\lambda_0, \quad \bar{t} = t\sqrt{gh_0}\lambda_0, \quad \bar{u} = \tfrac{3}{2}u(\varepsilon h_0)$$