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Strong convergence theorems on an iterative method for a family of finite nonexpansive mappings in reflexive Banach spaces ☆

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Abstract

In this paper, by using some new analysis techniques, we study the approximation problems of common fixed points of Halpern's iterative sequence for a class of finite nonexpansive mappings in strictly convex and reflexive Banach spaces by using Banach's limit. The main results presented in this paper generalize, extend and improve the corresponding results of Bauschke [The approximation of fixed points of compositions of nonexpansive mappings in Hilbert spaces, J. Math. Anal. Appl. 202 (1996) 150–159], Halpern [Fixed points of nonexpansive maps, Bull. Am. Math. Soc. 73 (1967) 957–961], Shioji and Takahashi [Strong convergence of approximated sequences for

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Let X be a real Banach space and X^* be the dual space of X. Let $\varphi : [0, \infty] =: R^+ \to R^+$ be a continuous strictly increasing function such that $\varphi(0) = 0$ and $\varphi(t) \to \infty$ as $t \to \infty$. This function φ is called a *gauge function*. The *duality mapping* $J_\varphi : X \to X^*$ associated with a gauge function φ is defined by

$$J_\varphi(x) = \{f \in X^* : \langle x, f \rangle = \|x\| \cdot \varphi(\|x\|), \|f\| = \varphi(\|x\|)\}, \quad x \in X,$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing between X and X^*. In the case that $\varphi(t) = t$, we write J for J_φ and call J the *normalized duality mapping*.

Now we give some elementary definitions:

Definition 1. A Banach space X is said to have a *weakly continuous duality mapping* if there exists a gauge function φ such that J_φ is single-valued and weak to weak star sequentially continuous.

It is known that $l^p (1 < p < \infty)$ has a weakly continuous duality mapping with a gauge function $\varphi(t) = t^{p-1}$. Setting

$$\Phi(t) = \int_0^t \varphi(\tau)\, d\tau, \quad t \geqslant 0,$$

then one sees that Φ is a convex function and

$$J_\varphi(x) = \partial \Phi(\|x\|), \quad x \in X,$$

where ∂ denotes the subdifferential in the sense of convex analysis. The subdifferential inequality

$$\Phi(\|y\|) \geqslant \Phi(\|x\|) + \langle y - x, j_x \rangle, \quad x, y \in X, \ j_x \in J_\varphi(x),$$

implies that the inequality

$$\Phi(\|x + y\|) \leqslant \Phi(\|x\|) + \langle y, j_{x+y} \rangle, \quad x, y \in X, \ j_{x+y} \in J_\varphi(x + y).$$