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Explicit travelling wave solutions of variants of the $K(n,n)$ and the $ZK(n,n)$ equations with compact and noncompact structures

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Abstract

In this work two powerful schemes, that use the reliable ideas of the sine–cosine method and the tanh method, are presented. Variants of the $K(n,n)$ and the $ZK(n,n)$ are selected to illustrate the two methods and to derive compact and noncompact solutions for these nonlinear variants with dispersive effects. The coefficients of the derivatives of the equation play a major role in change of the physical structures of the solutions.

Keywords: $K(n,n)$ equation; The KP equation; The ZK equation; Compactons; Sine–cosine method; The tanh method

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1. Introduction

The balance between the nonlinear convection uu_x and the linear dispersion u_{xxx} in the integrable nonlinear KdV equation

$$u_t + auu_x + bu_{xxx} = 0, \tag{1}$$

gives rise to solitons: waves with infinite support. Solitons are defined as localized waves that propagate without change of its shape and velocity properties and stable against mutual collisions [1–7].

Two well-known generalizations of the KdV equations, namely the integrable Kadomtsov–Petviashivilli (KP) equation, and the nonintegrable Zakharov–Kuznetsov (ZK) equation, given by

$$\{u_t + auu_x + u_{xxx}\}_x + ku_{yy} = 0 \tag{2}$$

and

$$u_t + auu_x + (\nabla^2 u)_x = 0, \tag{3}$$

respectively, were developed in [8,9], respectively, where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the isotropic Laplacian [9–12].

The delicate interaction between nonlinear convection $(u^n)_x$ with the genuine nonlinear dispersion $(u^n)_{xxx}$ in the well-known $K(n,n)$ equation [13]

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0, n > 1, \tag{4}$$

generates the so termed *compactons*: solitary waves with exact compact support. Compactons are defined as solitons with finite wavelengths or solitons free of exponential tails [13–22]. The solitary wave with compact support is called *compacton* to indicate that it has the property of a particle, such as phonon, photon, and soliton.

The stability analysis has shown that compacton solutions are stable, where the stability condition is satisfied for arbitrary values of the nonlinearity parameter. The stability of the compactons solutions was investigated by means of both linear stability and by Lyapunov stability criteria as well.

It is the objective of this work to further complement our studies in [20,21] on the $K(n,n)$ equation. Our first interest in the present work being in implementing the tanh method [22–24] to stress its power in handling nonlinear equations so that one can apply it to models of various types of nonlinearity. The next interest is the determination of exact travelling wave solutions with distinct physical structures to the $K(n,-n,2n)$ equation given by

$$u_t + a(u^n)_x + b[u^{-n}(u^{2n})_{xx}]_x = 0, \tag{5}$$

and the $K(n,2n,-n)$ given by

$$u_t + a(u^n)_x + b[u^{2n}(u^{-n})_{xx}]_x = 0. \tag{6}$$