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APPLIED MATHEMATICS AND COMPUTATION
ELSEVIER Applied Mathematics and Computation 173 (2006) 251–264
www.elsevier.com/locate/amc

Fuzzy compromise programming for portfolio selection

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Abstract

The aim of this paper is to solve a portfolio selection problem using Sharpe's single index model in a soft framework. Estimations of subjective or imprecise future beta for every asset can be represented through fuzzy numbers constructed on the basis of statistical data and the relevant knowledge of the financial analyst; the model, therefore, works with data that contain more information than any classical model and dealing with it does not involve a great extra computational effort. In order to solve the portfolio selection problem we have formulated a Fuzzy Compromise Programming problem. For this task we have introduced the fuzzy ideal solution concept based on soft preference and indifference relationships and on canonical representation of fuzzy numbers by means of their α-cuts. The accuracy between the ideal solution and the objective values is evaluated handling the fuzzy parameters through their expected intervals and using discrepancy between fuzzy numbers in our analysis. A major feature of this model is its sensitivity to the analyst's opinion as well as to the decision-maker's preferences. This allows interaction with both when it comes to design the best portfolio.

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doi:10.1016/j.amc.2005.04.003

Keywords: Portfolio selection; Fuzzy sets; Fuzzy compromise programming; Discrepancy between fuzzy numbers

1. Introduction

Portfolio selection has been one of the most important research fields in modern finance. In this context Markowitz mean–variance model [1] has been unanimously recognized as a pioneering work (see [2] for a brief historical account). A lot of models and extensions have been proposed to improve the performance of portfolio investment which have led to a great number of papers, monographs, and textbooks with the aim of formulating risk and returns of economic agents and understanding diversification in investment strategies [3].

Markowitz's portfolio selection model presents two main difficulties for being applied. First one, data required. If we could accurate expectations about future mean returns for each asset and the correlation of returns between each pair of assets then, the Markowitz's model under certain conditions and supposed known the investors' utility function, would produce optimum portfolios. The obtaining of accurate forecast of input data needed for this model is a difficult task, particularly in the case of the variance–covariance matrix between securities [4]. Second one, there is a computational difficulty associated to the resolution of large-scale quadratic programming problems with a dense covariance matrix. Several efforts to transform the quadratic problem into a linear one have been outlined in [5]. These authors have proposed a measure L_1 of risk instead of a L_2 measure of the Markowitz's model and then they have formulated a linear problem. If it seems to be not convenient or not possible to reduce the quadratic initial problem into a linear one, other solving methods as simulate annealing, tabu search and genetic algorithm, have been developed in order to solve the problem [6,7].

In this paper the portfolio selection problem has been handled in a soft framework. In this sense, this work can be regarded as a contribution to the determination of risk handling imprecise information in Beta estimation. Imprecision will be quantified by means of fuzzy numbers that represent the continuous possibility distributions for fuzzy parameters and hence place a constraint on the possible values the parameter may assume. In a previous work, [8] the authors have proposed a Goal Programming model including imprecise investor's aspirations concerning assets proportions of both, high and low risk assets. Semantics of these goals are based on the fuzzy membership of a goal satisfaction set. In this work, instead satisfying solutions we have obtained a compromise set of efficient solutions which allow the Decision