Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 173 (2006) 616–635
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

CONSLAW: A Maple package to construct the conservation laws for nonlinear evolution equations

Ruo-Xia Yao [a,b,*], Zhi-Bin Li [b]

[a] *Department of Computer Science, Shanghai Normal University, Shanghai 200234, PR China*
[b] *Department of Computer Science, East China Normal University, Shanghai 200062, PR China*

Abstract

An improved algorithm to explicitly compute the polynomial-type conservation laws for nonlinear evolution equations (either uniform in rank or not) is introduced and a software package CONSLAW written in Maple to automate the computation is developed. CONSLAW can construct the polynomial-type conservation laws for polynomial partial differential equations automatically. Furthermore, some new integrable systems can be filtered out by analyzing the compatibility conditions, which guarantee the existence of the conservation laws for a given parameterized nonlinear evolution equations. The explicit forms of the conserved densities play an important role in studying the integrability, such as explicit analytical solutions, bi-Hamiltonian form, one-parameter family of Bäcklund transformations, Lax pairs, and the checking of the accuracy of numerical integration algorithm. The effectiveness of CONSLAW is illustrated by applying it to a variety of equations.

* Corresponding author. Address: Department of Computer Science, Shanghai Normal University, Shanghai 200234, PR China.
E-mail addresses: RXYA02@hotmail.com (R.-X. Yao), lizb@cs.ecnu.edu.cn (Z.-B. Li).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.04.049

Keywords: Nonlinear evolution equation; Scaling symmetry; Conservation laws; Symbolic computation

1. Introduction

Over the past two decades, the most two famous discoveries in nonlinear physics are the soliton and the so-called "chaos theory", which, incidentally, were made with the aid of computer experiments, have radically changed the thinking of scientists about the nature of nonlinearity. It was in 1895 that Korteweg and de Vries derived the equation for water waves in shallow channels, which confirmed the existence of solitary waves. The stability and particle-like behavior of the soliton solutions could only be explained by the existence of many conservation laws, and as is well known, the obtained explicit forms of the conservation laws (CLaws) of the celebrated KdV equation led to the discovery of the Miura transformation, given by Miura and Gardner, and the discovery of the Lax pair. Maybe the most important contribution of the discovery of the explicit forms of CLaws was the development of the famous inverse scattering method, which, up to now, has been proved to be one of the most effective method for solving nonlinear partial differential equation or equations (PDEs). The existence of an infinite number of CLaws indicates the integrability of PDEs, although the nonexistence of a sequence of CLaws does not preclude the integrability, for example, the Burgers equation. Furthermore, it could make the construction of the bi-Hamiltonian form become easier for some important nonlinear evolution equations. A very extensive study of CLaws is found in [1] which includes both Lagrangian and Hamiltonian formulations. As shown that the work of the construction of the explicit forms of CLaws is meaningful, which can expedite the process of the study of nonlinear PDEs. For the first few CLaws, one can obtain them by hand. However, with the increase of the degree of the conserved densities, the construction by hand becomes harder and harder and even impossible. It is exciting that the great advantage of searching for polynomial-type conservation laws (PCLaws) lies in that they can be found by explicit computation. As a matter of fact, possessing such PCLaws is an intrinsic and common property for most of the nonlinear evolution equations. Furthermore, from the viewpoint of CLaws, a PDE is said to be "C-integrable" if it possesses infinitely many or lots of CLaws. The most important thing therefore is how to discover them. So, several symbolic programs have been developed for this task on the different platforms of symbolic computation systems. Sanders et al. developed a software package in `Maple` and `FORM` [2,3]. They use an extension of the total derivative operator to a Heisenberg algebra, which allows them to invert the total derivatives on its image. Thomas Wolf designed a `REDUCE` package ConLaw1-4 [4]. His