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A new nonmonotone spectral residual method for nonsmooth nonlinear equations[☆]

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ABSTRACT

In this paper, a new spectral residual method is proposed to solve systems of large-scale nonlinear equations, where the steplength is obtained by minimizing the residue of an approximate secant equation. Especially, the new steplength can be directly applied into solving strictly convex quadratic function. Combined with a new nonmonotone line search strategy, a new derivative-free algorithm, called a nonmonotone spectral residual algorithm (NSRA), is developed. Under mild assumptions, global convergence is established for locally Lipschitz continuous nonlinear systems. Compared with the state-of-the-art algorithms available in the literatures, the new algorithm is more efficient in solving large-scale benchmark test problems.

1. Introduction

Consider a system of nonlinear equations:

$$F(x) = 0, \tag{1.1}$$

where $F : R^n \to R^n$ is a locally Lipschitzian function, but it is non-smooth. In real applications, many equilibrium problems from economics, management sciences, engineering and mechanics can be described by variational inequalities and complementarity problems, which are often cast into (1.1). Owing to the importance and difficulty in solving the non-smooth nonlinear system (1.1), development of efficient algorithms has attracted great attentions of many researchers from the relevant scientific and engineering fields.

One of the most popular numerical methods for solving (1.1) is to construct an iterative format as follows:

$$x_{k+1} = x_k + \alpha_k d_k, \tag{1.2}$$

where α_k is called a steplength, d_k represents a search direction, and $\{x_k\}$ is a desired sequence of approximate solutions.

In the case that F is smooth, the Newton-type and quasi-Newton-type methods are the most popular ways to generate the search direction for solving (1.1) (see [1,2]). Though these methods are quite well known owing to their attractive

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local convergence, it is inappropriate to solve large scale nonlinear equations since it is difficult to compute and store the (approximate) Jacobian matrices of F (see [3,4]). In addition, the search directions in these methods are associated with solving large scale system of linear equations. To overcome the drawbacks in the Newton-type or quasi-Newton-type methods, some spectral gradient methods (see [5–8]) and conjugate gradient methods (see [9]) have been extended to solve (1.1). Numerical experiments have shown their advantage of numerical efficiency in solving large-scale nonlinear system of equations. In particular, $d_k = -F(x_k)$ is often regarded to be the most direct and simplest search direction due to its cheap computation. In the case that F is only locally Lipschitz (a special case of non-smoothness), both smoothing and smoothing-free methods exist for solving (1.1) (see, for example, [10,11]).

As for the steplength, many rules of monotone and non-monotone line search have been investigated in the literatures. For the recent results, one can see [12,2,13] and the references therein. For large scale unconstrained optimization problems, it is noted that the so-called Barzilai and Borwein steplength proposed in [14] has been acclaimed to be helpful to generate a sequence of approximate solutions, which is similar to those obtained by the quasi-Newton method. Specifically, consider the following unconstrained optimization problem:

$$\min_{x\in R^n} f(x),$$

where $f : R^n \to R$ is continuous differentiable, the so-called Barzilai and Borwein steplength is given by

$$\alpha_k = \arg\min_{\alpha} \|\alpha y_{k-1} - s_{k-1}\|^2, \tag{1.3}$$

or by

$$\alpha_k = \arg\min_{\alpha} \|\alpha^{-1} s_{k-1} - y_{k-1}\|^2. \tag{1.4}$$

Actually, from (1.3) and (1.4), we know that the Barzilai and Borwein steplength (BBS) is calculated by

$$\alpha_k = \frac{s_{k-1}^T y_{k-1}}{y_{k-1}^T y_{k-1}}, \tag{1.5}$$

or

$$\alpha_k = \frac{s_{k-1}^T s_{k-1}}{s_{k-1}^T y_{k-1}}, \tag{1.6}$$

where $s_{k-1} = x_k - x_{k-1}$ and $y_{k-1} = \nabla f(x_k) - \nabla f(x_{k-1})$. In [5–8], the above Barzilai and Borwein method has been extended to solve smooth nonlinear equations.

In this paper, we intend to propose a new spectral steplength such that it is more suited for solving large scale nonlinear system of equations. Actually, it will be shown that our steplength is exactly the geometrical mean of two types of Barzilai and Borwein steplengths. To demonstrate the advantages of the new steplength, our another focus is to develop a novel nonmonotone spectral residual algorithm for solving the nonsmooth nonlinear equations (1.1), where the new steplength is incorporated into designing of a nonmonotone line search.

The remainder of the paper is organized as follows. In next section, we will present the new spectral steplength and apply it into solving the quadratical minimization problems. In Section 3, we will develop a new nonmonotone spectral residual method for the general nonsmooth nonlinear system of equations and establish the convergence theory. We will report the numerical results of the new algorithm in Section 4. Some conclusions are drawn in Section 5.

2. New spectral steplength and its application in quadratical minimization

In this section, we will propose a new spectral steplength, which is suited for the nonlinear equations. Then, we apply the obtained steplength into solving the quadratical minimization problems.

We first address how to construct a new steplength for solving a system of nonlinear equations.

Let us consider the following system of linear equations:

$$Ax = b, \tag{2.1}$$

where A is symmetric positive definite. If we define $L : R^n \to R^n$, $L(x) = Ax - b$. Then, (2.1) can be rewritten as

$$L(x) = 0. \tag{2.2}$$

It is clear that $x_* \in R^n$ is a solution of (2.2) if and only if x_* is an optimal solution of the following quadratical minimization problem:

$$\min q(x) = \frac{1}{2}x^T Ax - b^T x, \quad x \in R^n. \tag{2.3}$$

For any given two points $x_k, x_{k-1} \in R^n$, denote

$$s_{k-1} = x_k - x_{k-1}, \qquad y_{k-1} = L(x_k) - L(x_{k-1}).$$