Journal of Computational and Applied Mathematics 313 (2017) 259–272

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Pricing vulnerable path-dependent options using integral transforms

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ARTICLE INFO

Article history:
Received 10 August 2015

Keywords:
Vulnerable barrier option
Vulnerable double barrier option
Vulnerable lookback option
Method of image
Double Mellin transform

ABSTRACT

In the over-the-counter (OTC) markets, the holders of many contracts are vulnerable to counterparty credit risk. Because of this issue, vulnerable options must be considered. In addition, in a financial environment, the pricing of path-dependent options yields many interesting mathematical challenges. In this paper, we study the pricing of vulnerable path-dependent options using double Mellin transforms to investigate an explicit (closed) form pricing formula or semi-analytic formula in each path-dependent option.

1. Introduction

Since the global financial crisis of 2007–2008, concerns have quickly increased regarding financial derivatives that are subject to credit default risk in over-the-counter (OTC) markets. In general, because of the unstructured stock exchange in the OTC markets, the option holder is always prone to counterparty credit risks because the option writer of the counterparty may not make the promised payments at the exercise date. We call these options that are subject to the default risk vulnerable options. Certainly, the value of the option that is vulnerable to the counterparty credit risk is less than an identical non-vulnerable option, because of the possibility of default. The pricing of European vulnerable options was first proposed by Johnson and Stulz [1] in 1987. Klein [2] developed results so that the counterparty can have other liabilities in the capital structure, and derived an analytic pricing formula based on the correlation between the underlying asset of the option and the option writer's assets. Dai and Chiu [3] used the first passage model to study the closed formula of the European vulnerable option. Meanwhile, Yang et al. [4] derived the pricing of vulnerable options under the stochastic volatility model.

In the paper, we discuss the pricing of vulnerable path-dependent options using a Mellin transform technique in cases of the vulnerable barrier option, the vulnerable lookback option, and the vulnerable double-barrier option. The lookback and barrier options, among many path-dependent options, are the most popular derivatives traded in stock exchanges around the world. In finance terminology, the barrier option is an exotic derivative that is typically an option on the underlying asset whose price hitting the predetermined barrier level either springs the option into existence or vanishes an already existing option. Lookback options are a type of exotic option with path dependency so that the payoff relies on the optimal (maximum or minimum) underlying asset price arising over the life of the option. The pricing of these path-dependent options is tricky and presents mathematical challenges because the value of the option at any time depends on the path taken by the underlying asset, as well as simply on the underlying asset at that point. There are many contributions on the analytic valuation of barrier and lookback options based on the Black–Scholes framework. The analytic solution for floating

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http://dx.doi.org/10.1016/j.cam.2016.09.024
0377-0427/

and fixed strike lookback options was derived by Goldman et al. [5] and Conze and Viswanathan [6]. Dai et al. [7] found an explicit closed solution for quanto lookback options. On the other hand, the analytic formula for a barrier option was first proposed by Merton [8]. Reiner and Rubinstein [9] then provided the closed formula for all eight types of barriers with a cumulative normal distribution function.

The Mellin transform is an integral transform that is considered as the multiplicative version of the two-sided Laplace transform. Many researchers have mostly induced the analytic formula for the evaluation of options by using probabilistic techniques. However, the introduction of the analytic approach using the Mellin transform enables us to resolve the complexity of the calculation of the options with probabilistic methods. In fact, Panini and Srivastav [10] obtained the pricing formula of European and American vanilla and basket options using the Mellin transform. Frontczak [11] found the pricing formula of options with jump diffusion models by using the Mellin transform technique. In addition, Yoon and Kim [12] derived European vulnerable options under stochastic (Hull White) interest rates, as well as under a constant interest rate by the double Mellin transform, Yoon [13] used the Mellin transform to compute a closed-form formula for Europeans in a Black–Scholes setting with a stochastic interest rate, and Jeon et al. [14] used the techniques of the double Mellin transform to obtain the explicit solution of time-dependent coefficients Black–Scholes partial differential equation and then find a closed-form formula of vulnerable geometric Asian options from the explicit solution. Moreover, Jeon et al. [15] investigated a general solution for the inhomogeneous Black–Scholes partial differential equation with mixed boundary conditions using Mellin transform techniques to price the Russian options with finite time horizon.

In addition, to solve the problem of the vulnerable path-dependent options, the method of image is a very important tool. Therefore, we have to consider not only the Mellin transform method but also the method of images mentioned in [16] for the pricing. The method of images is closely related to the reflection principle of the expectation solution. Using the partial differential equation (PDE) method of images allows us to derive the pricing formula of path-dependent options more easily compared to the existing method. That is, by using the method of images, we transform the PDE of the barrier or lookback option with two conditions (boundary and final condition) into the PDE with the final condition of the enlarged range of underlying assets, and then find the pricing formula of the options using Mellin transform techniques. Buchen [16] derived the pricing formula of barrier options more easily than the existing method using the method of images, and Buchen and Konstandatos [17] investigated the pricing double-barrier options with the identical method. Jeon and Yoon [18] and Jeon et al. [19] derived the value of the lookback option price and the pricing formula of the double-barrier option under stochastic volatility model, respectively, using the Mellin transform and the method of images.

This paper is organized as follows. In Section 2, we derive the price of a vulnerable barrier put option with a down-and-out type. Section 3 considers a vulnerable floating strike lookback put option and then obtains the analytic formula of the option for the given PDE. Section 4 applies the method of images and the Mellin transform to induce a closed-form analytic solution for the double barrier option. Section 5 discusses the concluding remarks.

2. Vulnerable barrier option

In this section, we examine the price of a vulnerable barrier option of a down-and-out type. To derive an explicit closed solution, we exploit the method of image and a double Mellin transform.

2.1. Model formulation

Let X_t be an underlying asset value with a constant drift rate of the underlying asset, μ_x, and a volatility, σ_x. Moreover, let V_t be the value of the assets of the counterparty (the option writer) with drift rate μ_v and volatility σ_v. We then consider an underlying asset price model given by the stochastic differential equation

$$dX_t = \mu_x X_t dt + \sigma_x X_t dW_t^x,$$
$$dV_t = \mu_v V_t dt + \sigma_v V_t dW_t^v,$$

where W_t^x and W_t^v are standard Brownian motions given by $d\langle W_t^x, W_t^v\rangle_t = \rho dt$, and σ_x and σ_v are positive constants. Under a risk-neutral probability measure, this model is changed by the SDEs:

$$\begin{aligned} dX_t &= rX_t dt + \sigma_x X_t dW_t^{x*}, \\ dV_t &= rV_t dt + \sigma_v V_t dW_t^{v*}, \end{aligned} \tag{2.1}$$

where r is a positive constant interest rate, and W_t^{x*} and W_t^{v*} are the transformed standard Brownian motions satisfying $d\langle W_t^{x*}, W_t^{v*}\rangle_t = \rho dt$.

Now, we set up the PDE of the vulnerable down-and-out barrier options with the boundary and terminal conditions. The final condition is determined by the payoff function, depending on the financial distress environment. At the time $t = T$, the payoff of a put option is given by

$$\tilde{h}(X_T, V_T) = (K - X_T)^+ \mathbf{1}_{\{\min_{0\le t\le T} X_t > B\}} \left(\mathbf{1}_{\{V_T \ge D^*\}} + \mathbf{1}_{\{V_T < D^*\}} \frac{(1-\alpha)V_T}{D} \right),$$