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Centers and limit cycles of polynomial differential systems of degree 4 via averaging theory

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ABSTRACT

In this paper we classify the phase portraits in the Poincaré disc of the centers of the generalized class of Kukles systems

$$\dot{x} = -y, \quad \dot{y} = x + ax^3y + bxy^3,$$

symmetric with respect to the y-axis, and we study, using the averaging theory up to sixth order, the limit cycles which bifurcate from the periodic solutions of these centers when we perturb them inside the class of all polynomial differential systems of degree 4.
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1. Introduction and statement of the main results

Two of the classical and difficult problems in the qualitative theory of polynomial differential systems in $\mathbb{R}^2$ is the characterization of their centers, and the study of the limit cycles which can bifurcate from their periodic orbits when we perturb them inside some class of polynomial differential equations.

Our work is related with the class of polynomial differential systems of the form

$$\dot{x} = -y, \qquad \dot{y} = x + Q_n(x, y), \tag{1}$$

having a center at the origin, where $Q_n(x, y)$ is a homogeneous polynomial of degree n, and in the study of the number of limit cycles which bifurcate from the periodic orbits of these centers when they are perturbed inside the class of all polynomial differential systems of degree n.

Differential polynomial systems (1) were called *Kukles homogeneous systems* in [1], see also [2,3]. The centers of systems (1) started to be studied by Volokitin and Ivanov in [4].

For $n = 1$ the differential systems (1) are linear, they can have centers, but the perturbation of these centers inside the class of linear differential systems cannot produce limit cycles, because it is well known that linear differential systems cannot have isolated periodic solutions in the set of all periodic solutions.

For $n = 2$ the phase portraits of system (1) symmetric with respect to the y-axis are a particular class of quadratic centers, and these are well studied, see [5].

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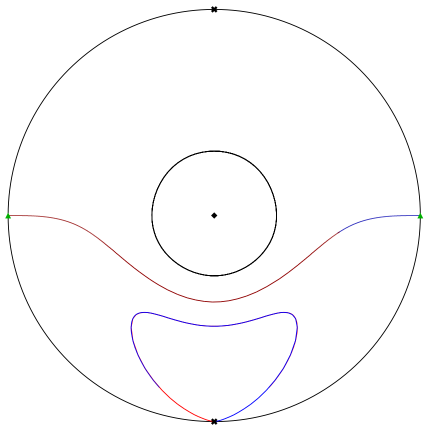

Fig. 1. Case $a > 0$ and $b = 0$. The separatrices of this phase portrait are the circle of the infinity; and an orbit A which connects the two separatrices inside the Poincaré disc of the two saddles at infinity, localized at the origins of the local charts U_1 and V_1. Therefore this phase portrait has two canonical regions. The canonical region limited by the orbit A and the part of infinity containing the origin of U_2 is filled by the periodic orbits of the center; and the canonical region limited by the orbit A and the part of infinity containing the origin of V_2 is filled by an elliptic sector of the infinite singular point localized at the origin of V_2.

In [6–10] are characterized the centers and the phase portraits of linear systems with homogeneous nonlinearities of degree 3, so in particular the phase portraits of systems (1) with $n = 3$. The limit cycles that bifurcate from the periodic orbits of the centers of systems (1) with $n = 3$ when they are perturbed inside the class of all cubic polynomial differential systems were studied inside the more general articles [11–13].

Giné in [1] proved that for $n = 4$ system (1) has a center at the origin if and only if its vector field is symmetric about one of the coordinate axes.

The first objective of this paper is to study the phase portraits of the centers of systems (1) with $n = 4$ which are symmetric with respect to the y-axis, i.e. the phase portraits of the systems

$$\dot{x} = -y, \qquad \dot{y} = x + ax^3y + bxy^3. \tag{2}$$

The second objective is to study the limit cycles that bifurcate from the periodic solutions of the centers of systems (2) when they are perturbed inside the class of all quartic polynomial differential systems.

For the definition of the global phase portrait of a polynomial differential system in the Poincaré disc see Section 2, where we provide the notations, definitions and basic results which we need for reaching our two objectives.

Our first main result is the following.

Theorem 1. *A polynomial differential system (2) with $a^2 + b^2 \neq 0$ has a phase portrait in the Poincaré disc topologically equivalent to one of the three phase portraits of Figs. 1–3.*

Theorem 1 is proved in Section 4.

We write the perturbed quartic polynomial differential system of system (2) as

$$\begin{aligned} \dot{x} &= -y + \sum_{s=1}^{6} \varepsilon^s \sum_{0\le i+j\le 4} a_{ij}^{(s)} x^i y^j, \\ \dot{y} &= x + ax^3y + bxy^3 + \sum_{s=1}^{6} \varepsilon^s \sum_{0\le i+j\le 4} b_{ij}^{(s)} x^i y^j, \end{aligned} \tag{3}$$

where i and j are non-negative integers. For the definition of the averaging theory of order $k = 1, \dots, 6$ see Section 5, where we denote by f_k the kth average function. In what follows we state our second main result.

Theorem 2. *For $|\varepsilon| \neq 0$ sufficiently small the maximum number of small amplitude limit cycles of the differential system (3) bifurcating from the periodic solutions of the center (2) is*

(a) 0 *if the first order average function f_1 is non-zero,*
(b) 0 *if $f_1 = 0$ and the second order average function f_2 is non-zero,*
(c) 1 *if $f_1 = f_2 = 0$ and the third order average function f_3 is non-zero,*
(d) 1 *if $f_1 = f_2 = f_3 = 0$ and the fourth order average function f_4 is non-zero,*

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