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Identification of time-dependent source terms and control parameters in parabolic equations from overspecified boundary data

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ABSTRACT

This paper presents a semigroup approach for inverse source problems for the abstract heat equation, when the measured output data is given in subject to the integral overspecification over the spatial domain. The existence of a solution to the inverse source problem is shown in appropriate function spaces and a representation formula for the solution is proposed. Such representation permits the derivation of sufficient conditions for the uniqueness of the solution. Also an approximation method based on the optimal homotopy analysis method (OHAM) is designed, and the error estimates are discussed using graphical analysis. Moreover, we conjecture that our approach can be applied for the determination of a control parameter in an inverse problem with integral overspecialization data. The proposed algorithm is examined through various numerical examples for the reconstruction of continuous sources and the determination of a control parameter in parabolic equations. The accuracy and stability of the method are discussed and compared with several finite-difference techniques. Computational results show efficiency and high accuracy of the proposed algorithm.

1. Introduction

Let us consider a Banach space X, a linear operator A in X, $z \in X$, $g \in C^1([0,\tau];\mathbb{R})$, and a linear functional $\phi \in X^*$. We study the inverse problem of finding a pair of functions $u \in C^1([0,\tau];X)$ and $p \in C([0,\tau];\mathbb{R})$ from the set of relations

$$u'(t) = Au(t) + p(t)z, \quad 0 \le t \le \tau, \tag{1}$$

$$u(0) = u_0, \tag{2}$$

$$\phi[u(t)] = g(t), \quad 0 \le t \le \tau. \tag{3}$$

Mathematical models related to inverse problems of this type arise in various physical and engineering settings such as, the identification of water sources and air pollution in the environment, or the determination of heat sources in heat conduction.

Heat source identification problems are the most commonly encountered inverse problems in heat conduction. These problems have been studied for several decades due to their significance in a variety of scientific and engineering

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applications (see [1–11]). In many heat conduction and diffusion problems, the source terms are unknown and usually are not easy to be detected directly. Hence, only one of the following typical measured output data is available and feasible from experiments:

$$\begin{cases} \int_0^l u(x,t)k(x)\,dx = g(t), \\ \quad u(x_0,t) = g(t). \end{cases}$$

These data are defined to be overspecified boundary (measured) data, according to inverse problems terminology.

The first attempt to study source identification problems for the time independent source $p(t)z \in X$, with the final overdetermination $u_T(x) := u(x,T)$, by the semigroup approach has been given in [12], where it is proved that when the elliptic operator $-A$ is positive definite and self-adjoint, the solution (u,p) of the source identification problem exists and is unique. A general representation formula for a solution of the source identification problem for the abstract parabolic equation $u_t(t) = Au(t) + F(t)$, was proposed in [9]. Note that an inverse source problem with final overdetermination for the one dimensional heat equation has first been considered by Tikhonov [13] in the study of geophysical problems. A semigroup approach for inverse source problems for the abstract heat equation $u_t = Au + F$, when the measured output data is given in the form the final overdetermination $u_T(x) := u(x,T)$ has been proposed in [11]. In this work abstract parabolic equation with overspecified boundary data is studied in half-plane and the uniqueness of the solution is proved. For parabolic equations in a bounded domain, various aspects of inverse source problems were studied in [14–16], etc.

Numerical methods for solving inverse problems for parabolic equations are considered in many works. Backward Euler approximation method has been introduced in [17] for the inverse problem of identifying a time dependent unknown coefficient in a parabolic problem subject to initial and non-local boundary conditions along with an overspecified condition defined at a specific point in the spatial domain. The idea in [18] is to change the problem of identifying an unknown time-dependent source term in an inverse problem of parabolic type with nonlocal boundary conditions to a system of Volterra integral equations and then to solve the system by means of a collocation method. Inverse problem of reconstructing the coefficient q in the parabolic equation $u_t - \Delta u + q(x)u = 0$ from the final measurement $u(x,T)$ has been solved using the optimization method combined with the finite element method in [19]. Recently, a numerical method depends on the Fourier regularization method for solving ill-posed problems of heat equation has been proposed in [20].

In comparison with previous studies on the subject of time-dependent heat source identification, we consider the case when A is a generator of a c_0-semigroup which is more general than [21,22] who considered only the case where A is a bounded linear operator and [23] who considered A as a generator of a c_0-semigroup with ϕA bounded. Our result in this case will be presented in Theorem 2 where much of detailed proof is completely different from the last mentioned studies. Moreover, we shall also consider the problem of finding a control parameter $p(t)$ in the following form

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + p(t)u + g(x,t), \quad 0 \le x \le 1,\ 0 < t \le T, \tag{4}$$

with the initial condition

$$u(x,0) = f(x), \quad 0 \le x \le 1, \tag{5}$$

and boundary conditions

$$u(0,t) = g_1(t), \quad 0 < t \le T, \tag{6}$$

$$u(1,t) = g_2(t), \quad 0 < t \le T, \tag{7}$$

with an additional condition which describes the overspecification over a portion of the spatial domain

$$\int_0^1 k(x)u(x,t)dx = E(t), \quad 0 < t \le T, \tag{8}$$

which has never been investigated analytically and it represents one of the main contributions of the present study. Some of the ideas from the proof of Theorem 2 are combined to form a numerical method based on the OHAM of the underlying inverse problems.

The paper is organized as follows. Section 2 gives the mathematical analysis of the inverse problem and recalls some previous perturbation results for linear operators in Banach spaces. Moreover, it introduces a quasisolution of the inverse source problems (1)–(3) based on the solution to the corresponding direct problem. Sections 3 and 4 describe the optimal homotopy analysis method for solving analytically the inverse source and control problems with overspecified boundary data observations; it also discusses the numerical results. Section 5 gives the conclusion of the paper and possible future work.

2. Mathematical analysis

To solve the problem (1)–(3) when the operator A is the infinitesimal generator of a c_0-semigroup on X, we propose a method coupled with the perturbation theory of linear operators. The proposed method, requiring no conditions such as ϕA is bounded, can eliminate the restrictions of the traditional methods.