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An isogeometric boundary element method for three dimensional potential problems

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ABSTRACT

Isogeometric analysis (IGA) coupled with boundary element method, i.e. IGABEM, received a lot of attention in recent years. In this paper, we extend the IGABEM to solve 3D potential problems. This method offers a number of key improvements compared with conventional piecewise polynomial formulations. Firstly, the models for analysis in the IGABEM are exact geometrical representation no matter how coarse the discretization of the studied bodies is, thus the IGABEM ensures that no geometrical errors are produced in the analysis process. Secondly, a meshing process is no longer required, which means redundant computations are eliminated to allow analysis to be carried out with greatly reduced pre-processing. To accurately evaluate the singular integrals appearing in our method, the power series expansion method is employed. The integration surface is on the real surface of the model, rather than the interpolation surface, i.e. no geometrical errors. Thus, the value of integral is more accurate than the traditional boundary element method, which can improve the computation accuracy of the IGABEM. Some numerical examples for 3D potential problems are used to validate the solutions of the present method with analytical and numerical solutions available.

1. Introduction

Isogeometric analysis (IGA), established by Hughes et al. [1], is a new numerical method that has received a lot of attention over the last ten years. The central idea of the method is to bridge the gap between computer aided design (CAD) and finite element analysis (FEA) by applying basis functions that are typically used in computer aided design (CAD) to the analysis, instead of the conventional polynomial basis functions. The main benefit of isogeometric method is that the geometry of the problem is preserved exactly. When building the models using CAD software, we can directly carry out numerical analysis based on the isogeometric method, i.e. there is no transformation from the CAD models to the computational meshes. In addition, in the implementation of the IGA the mesh refinement can be highly simplified using the standard knot-insertion and/or degree-elevation procedures [2] without communicating with the CAD system, once the initial mesh is completed.

The IGA was originally presented to focus on the use of the finite element method (FEM), but many researchers also applied the IGA to the boundary element method (BEM) where distinct advantages such as reduction of problem dimensionality by one are found. The IGA coupled with the BEM, i.e. IGABEM, has achieved rapid development recently [3–5] and has been successfully applied to investigate various problems, e.g. elasticity problems [6,7], potential problems [8], Laplace equation [9], fast multipole IGABEM [10,11], Helmholtz problems [12], ship wave-resistance problem [13,14], acoustic problems [15,16], shape optimization [17–19], nonsingular IGABEM analysis [20], weakly-singular integral equation [21], crack problem [22] and adaptive mesh-refinement [23].

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Compared with traditional BEM, the models for analysis in the IGABEM are exact geometrical representation no matter how coarse the discretization of the studied bodies is, thus the IGABEM ensures that no geometrical errors appear in the analysis process. This is a distinct advantage over traditional BEM, in which the geometrical errors will greatly influence the accuracy of the numerical result, especially for complex models or thin-body/coating structures. In addition, as we have known that in the implementation of the IGAFEM the information of the domains is needed which is still a challenging problem. However, the IGABEM only requires a boundary description of the problem which creates a perfect match with the CAD, since the output of such software is only a boundary discretization. Moreover, when the IGABEM is used to shape design optimization problems, it can provide more accurate sensitivity of complicated geometries including higher order effects such as curvature, normal and tangential vectors. Because the NURBS functions of higher continuity offer a much more compact representation of response and sensitivity of structures than the standard basis functions do, so it can yield better accuracy even at the same polynomial order.

However, the IGABEM formulation contains varied orders of singular integrals, which requires careful study and analysis. Up to now, tremendous effort has been devoted to deriving convenient integral forms or sophisticated computational techniques to eliminate the singularities appearing in boundary integral equations. These methods can be summarized on the whole as two categories: the local and the global strategies. The local strategies are employed to calculate the singular integrals directly. They usually include but are not limited to, analytical and semi-analytical methods [24,25], new Gaussian quadrature approach [26], the local regularization method [27,28], transformation method [29–31], finite-part integral method [32,33], subtraction technique [34,35], etc. Among these methods, the local regularization technique proposed by Guiggiani et al. [36,37] was extensively employed to remove various orders of singularities. Although the method was successfully used to evaluate the strong singular integral appearing in 2D IGABEM [5,6], it requires the expansion of all quantities of the integrand into Taylor's series, which may be arduous and computationally expensive [38,39]. The power series expansion method presented by Gao [40] seems to be a more accurate and flexible method for evaluating 2D or 3D singular boundary integrals by expressing the non-singular part of a singular integrand as well as the global distance r as power series in the local distance ρ of the intrinsic coordinate system.

In this paper, the power series expansion method [40] is used to compute the singular integrals appearing in the 3D IGABEM. Firstly, the singular surface integral is divided into a line integral over the contour of the integration surface and a radial integral containing the singularities by the radial integration method (RIM) [41,42]. Then the singularities condensed in the radial integral are removed analytically or numerically by extracting the finite value parts from the power series expansions. Finally, the line integral with regular kernel functions can be computed by the standard Gaussian quadrature. Through the above procedure, singular integrals over isogeometric boundary elements can be evaluated with high accuracy, even when the order of singularity is high. One of the features of the method is that integration surface is on the real surface of the model rather than the interpolation surface, i.e. no geometrical errors. Thus, the value of integral is more accurate than the traditional boundary element method [40].

A brief outline of this paper is as follows. A short introduction to B-spline and NURBS is given in Section 2. In Section 3, the numerical implementation of the IGABEM for 3D potential problems is performed. Several numerical examples are given in Section 4 to verify the efficiency and accuracy of the present method. Finally, we present the conclusions for our work.

2. B-splines and NURBS

In this work, we focus our attention on the numerical implementation of the IGABEM for 3D potential problems. Thus only some conclusions are given, more details about the IGABEM can be found in [1,5,6,43].

2.1. B-spline and NURBS basis functions

B-spline basis functions of degree p are defined recursively with piecewise constants ($p = 0$)

$$N_{i,0} = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $1 \leq i \leq n$, n is the number of basis functions which form the B-spline. For $p = 1, 2, 3, \ldots$, they are defined by

$$N_{i,p}(\xi) = N_{i,p-1}(\xi)\left(\xi - \xi_i\right) / \left(\xi_{i+p} - \xi_i\right) + N_{i+1,p-1}(\xi)\left(\xi_{i+p+1} - \xi\right) / \left(\xi_{i+p+1} - \xi_{i+1}\right). \tag{2}$$

The B-spline can be considered as a sub-set of NURBS. NURBS basis functions are defined as

$$R_{i,p}(\xi) = N_{i,p}(\xi)\omega_i \bigg/ \left(\sum_{j=1}^{n} N_{j,p}(\xi)\omega_j\right) \tag{3}$$

where $N_{i,p}$ is the ith B-spline basis function of order p. ω_i is the ith positive weight. The values of ω_i depend on the type and shape of curves.

To define a B-spline or NURBS curve several other items are required [1,5,6,43], such as curve degree$p$, control points $\boldsymbol{P}$, knot vector $\mathbf{U}$.