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The joint distribution of the Parisian ruin time and the number of claims until Parisian ruin in the classical risk model

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ABSTRACT

In this paper we propose new iterative algorithm of calculating the joint distribution of the Parisian ruin time and the number of claims until Parisian ruin for the classical risk model. Examples are provided when the generic claim size is mixed Erlang distributed. We focus on the exponentially and the Erlang(k, μ) $(k \geq 2)$ distributed claim sizes.

1. Introduction

The distribution of the number of claims until ruin has been in the center of interest for many years. One of the first references dealing with this problem is Beard [1]. The main first step was done by Stanford and Stroiński [2] who produced recursive procedures to calculate the probability of ruin at the nth claim arrival epoch in the classical risk model. Egído dos Reis [3] derived the moment generating function of the number of claims until ruin in the classical risk model. He inverted this for certain claim size distributions, and, using a duality argument, found moments of the number of claims until ruin when the initial surplus is 0. The next main step was done by Landriault et al. [4] who considered a Sparre Andersen risk model with exponential claims. Using Gerber–Shiu type analysis (see Gerber and Shiu [5]) they derived a number of results including an expression for the probability function of the number of claims until ruin. The main idea of getting these nice results followed approach of Dickson and Willmot [6]. The main results of our paper are closely related with the seminal paper of Dickson [7] who using probabilistic arguments derived the expression for the joint density of the time of ruin and the number of claims until ruin in the classical risk model. From this he obtained a general expression for the probability function of the number of claims until ruin. He also considered the moments of the number of claims until ruin and illustrate all results in the case of exponentially distributed individual claims. Frostig et al. [8] and Zhao and Zhang [9] analyzed similar problems.

In this paper we extend results concerning classical ruin into so-called Parisian type of ruin. This type of ruin occurs if the surplus process falls below zero and stays below zero for a continuous time of interval of length d; see Fig. 1. We believe

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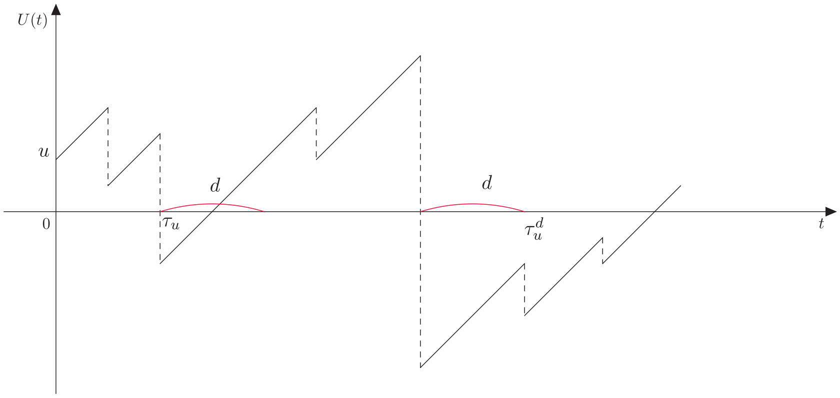

Fig. 1. Surplus process $U(t)$ and Parisian ruin.

that the Parisian ruin probability and other related quantities might be more appropriate measures of risk than the ones identified for the classical ruin. The main reason is that it gives the insurance companies the chance to achieve solvency. The idea of Parisian ruin comes from Parisian options which was first introduced by Chesney et al. [10]. Dassios and Wu [11] considered the Parisian ruin probability for the classical risk model with exponential claims and for the Brownian motion with drift. Czarna and Palmowski [12] and Loeffen et al. [13] analyzed the Parisian ruin probability for a general spectrally negative Lévy process. Other relevant papers are Landriault et al. [14,15], where the deterministic and fix delay d is replaced by an independent exponential random variable.

Our paper in a sense has similar goal like in Dickson [7] and Landriault et al. [14], that is we want to identify the joint density of the time of Parisian ruin and the number of claims until Parisian ruin. This density is interesting from the actuarial point of view since the Parisian delay as well as the number of claims appeared up to this ruin time recognize an insurer's capital insufficiency and as such is a very good risk measure. Although our focus is more consistent with Stanford and Stroiński [2] — we want to create efficient iterative algorithm of finding above quantity. Furthermore, in the last section of this paper, we present an example, where the claim amounts distribution is a mixture of Erlang distributions with the same parameter. This example is important from the application point of view, since this class of distributions is extremely flexible in terms of possible shapes of the probability density function of its members. It also includes many distributions whose membership in the class is not immediately obvious (such as the generalized Erlang distribution, exponential distribution etc.). For background on the mixed Erlang distributions we refer to seminal paper of Willmot and Lin [16].

Formally, in this paper we consider a continuous-time surplus process:

$$U(t) := u + ct - S(t), \tag{1}$$

where the non-negative constant u denotes the initial reserve, the positive constant c is the rate of premium income. The process $\{S(t)\}_{t\geq 0}$ is a compound Poisson process, with $S(t) = \sum_{i=1}^{N_t} X_i$, where N_t describes the number of claims counted up to time t which is a Poisson process with parameter λ and $\{X_i\}_{i=1}^{\infty}$ are claim sizes which are independent and identically distributed non-negative random variables that are also independent of N_t. We denote by $F(x)$ and $f(x)$ the distribution function and density function, respectively. We also adopt the notation $\overline{G}(x, t) = 1 - G(x, t) = \mathbb{P}(S(t) > x)$ with

$$g(x, t) = \frac{\partial}{\partial x} G(x, t). \tag{2}$$

We assume $c > \lambda \mathbb{E}(X_1)$ assuring that ruin is not certain.

We define the Parisian time of ruin by

$$\tau_u^d := \inf\{t > 0 : t - \sup\{s < t : U(s) \geq 0\} \geq d, U(t) < 0\}.$$

We denote the joint density of $N_{\tau_u^d}$ and τ_u^d by (hereafter $\mathbb{N} = \{0, 1, 2, \ldots\}$):

$$w_u^d(n, t) := \frac{d}{dt} \psi_u^d(n, t), \quad n \in \mathbb{N}, t \geq 0$$

with

$$\psi_u^d(n, t) := \mathbb{P}(N_{\tau_u^d} = n, \tau_u^d \leq t | U(0) = u), \quad n \in \mathbb{N}, t \geq d. \tag{3}$$

Further, let $p_u^d(n)$ denotes the probability that there have been exactly n claims up to Parisian ruin event, so that

$$p_u^d(n) := \mathbb{P}(N_{\tau_u^d} = n, \tau_u^d < \infty | U(0) = u) = \int_d^{\infty} w_u^d(n, t) dt. \tag{4}$$

The main goal of this paper is to give efficient iterative algorithm of calculating of $w_u^d(n, t)$ and hence $p_u^d(n)$.