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On the iterative learning control for stochastic impulsive differential equations with randomly varying trial lengths☆

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ABSTRACT

In this paper, a new class of stochastic impulsive differential equations involving Bernoulli distribution is introduced. For tracking the random discontinuous trajectory, a modified tracking error associated with a piecewise continuous variable by zero-order holder is defined. In the sequel, a new random ILC scheme by adopting global and local iteration average operators is designed too. Sufficient conditions to guarantee the convergence of modified tracking error are obtained by using the tools of mathematical analysis via an impulsive Gronwall inequality. Finally, two illustrative examples are presented to demonstrate the performance and the effectiveness of the averaging ILC scheme to track the random discontinuous trajectory.

1. Introduction

Iterative learning control (ILC) is a useful control method in terms of basic theory and experimental applications [1–5] by adjusting the past control experience again and again to improve the current tracking performance. In the past two decades, there are many contributions on ILC with uniform trial lengths for determinate type ODEs [6–12], FDEs [13–16] and PDEs [17–20] as well as other relevant formulations for control systems in literature, such as semilinear fractional evolution systems [21,22]. However, there are only few works discussing the ILC problems with non-uniform trial lengths in existing literature. It should be mentioned here that there are also a large number of contributions on impulsive differential systems [23–25]. In many applications of ILC, it will appear the case that every trial ends in some not fixed time of duration [26]. For example, when stroke patients walk on a treadmill, their steps will be usually cut short by suddenly putting the foot down or meeting random obstruction or perturbed by instantaneous impulses.

Recently, Li et al. [27] initially apply the iteration-average operator to the ILC problems with non-uniform trial lengths for discrete linear time-invariant systems, where the trial lengths randomly vary in the iteration domain. An interesting ILC scheme with an iteration-average operator is designed for tracking tasks with non-uniform trial lengths. However, to the

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best of our knowledge, there are no works dealing with ILC problems with non-uniform trial lengths for stochastic impulsive differential systems.

In this paper, we introduce a new class of stochastic impulsive differential equations involving Bernoulli distribution, defining a modified tracking error associated with a piecewise continuous variable by zero-order holder, and design a new random ILC scheme by adopting global and local iteration average operators. By using the tools of mathematical analysis via an impulsive Gronwall inequality, we present some sufficient conditions to guarantee the convergence of modified tracking error.

The paper is organized as follows. Section 2 introduces a new class of stochastic impulsive differential equations and random ILC design. Section 3 presents convergence result for random ILC updating law with non-uniform trial lengths. Section 4 gives two illustrative examples.

2. Stochastic impulsive differential equations and random ILC design

Inspired by [27, Definition 1], throughout this paper, we denote $\mathbf{E}\{X\}$ by the expectation of the stochastic variable X and $\mathbf{p}[g]$ by the occurrence probability of the event g. By Jensen's inequality [28, Theorem 1.6.2], one has $|\mathbf{E}\{X\}| \leq \mathbf{E}\{|X|\}$.

Define $\theta(i)$ by a stochastic variable in the ith phases. Let $\theta(i),\ i \in \{1, 2, 3, \ldots, N\}$ be a stochastic variable satisfying Bernoulli distribution and taking values 0 or 1.

Define a set $\gamma(i)$ as follows:

$$\gamma(i) = \begin{cases} \big[t_{i-1}, t_i\big) \subset \big[0, T\big], & \prod_{j=1}^{i} \theta(j) = 1, \quad i \in \{1, 2, 3, \ldots, N-1\}, \\ \big[t_{N-1}, T\big] \subset \big[0, T\big], & \prod_{j=1}^{N} \theta(j) = 1, \\ \emptyset, & \prod_{j=1}^{i} \theta(j) = 0, \quad i \in \{1, 2, 3, \ldots, N\}, \end{cases}$$

where t_i is satisfying $0 = t_0 < t_1 < \cdots < t_{N-1} < t_N = T$.

Consider a new class of random impulsive differential equations described as:

$$\begin{cases} \dot{x}(t) = ax(t) + f(t), \quad t \in \bigcup_{i=1}^{N} \gamma(i) \setminus \{t_1, t_2, t_3, \ldots, t_{N-1}\}, \\ x(t_i^+) - x(t_i^-) = \rho_i(x(t_i^-)) + \varrho_i, \quad i \in \{1, 2, \ldots, N-1\}, \\ x(0) = x_0, \end{cases} \tag{1}$$

where $a, x(t), \varrho_i, x_0 \in \mathbb{R}, f : [0, T] \to \mathbb{R}$ is continuous, $\rho_i : \mathbb{R} \to \mathbb{R}$ is Lipschitz conditions, $x(t_i^+) := \lim_{\epsilon \to 0^+} x(t_i + \epsilon)$ and $x(t_i^-) := \lim_{\epsilon \to 0^-} x(t_i + \epsilon)$ represent the right and left limits of $x(t)$ at $t = t_i$ respectively.

Let $J \subset \mathbb{R}$ and $C(J, \mathbb{R})$ be the set of all continuous functions from J into $\mathbb{R}$. We introduce the piecewise continuous functions space $PC(J, \mathbb{R}) := \{x : J \to \mathbb{R} \mid x \in C((t_i, t_{i+1}], \mathbb{R}),\ i = 0, 1, \ldots, N$ and $\exists x(t_i^-)$ and $x(t_i^+),\ i = 1, \ldots, N,$ with $x(t_i^-) = x(t_i)\}$ endowed with λ-norm $\|x\|_\lambda = \sup_{t \in J} |x(t)|e^{-\lambda t}$.

One can use standard methods [29, Theorem 2.1] to derive that the problem (1) has a unique solution $x \in PC([0, T], \mathbb{R})$ given by

$$x(t) = e^{at}x_0 + \int_0^t f(s)ds + \sum_{0<t_j<t} e^{a(t-t_j)}[\rho_j(x(t_j^-)) + \varrho_j], \quad t \in \gamma(i) \neq \emptyset.$$

Or we write it directly as follows:

$$x(t) = x_0 + \int_0^t (ax(s) + f(s))ds + \sum_{0<t_j<t} [\rho_j(x(t_j^-)) + \varrho_j], \quad t \in \gamma(i) \neq \emptyset.$$

In order to make the reader understand the solutions to the problem (1), we list the following figures.