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Statistical analysis of solution accuracy for inverse problems in electrodynamics

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ABSTRACT

Calculation of electric/magnetic field parameters and source intensity based on measurements is discussed. It is required to assess the accuracy of obtained solution. Upper estimates of the measurement error limits and analytical approaches to error calculations give considerable overestimations and prove to be inefficient for practical applications. The approach based on statistical estimate of solution error is proposed. The obtained error estimate is in good agreement with experimental data.

1. Introduction

Constant magnetic field and static electric field of various engineering objects are considered. Let us describe the field sources by function u considered as an element of normalized space $U \quad (u \in U)$, the measured field values (input data f) are considered as an element of the normed space $F \quad (f \in F)$. The relation between u and f is governed by the operator equation

$$Au = f, \tag{1}$$

where operator A continuously represents U in F.

Determination of sources u based on the known values of field f necessitates solution of an inverse problem. In some cases, it is useful to determine functionals based on distribution of sources: the problem for calculation of values is formulated

$$g = Cu, \tag{2}$$

where C is an operator acting from the normed space U to the space G.

For the magnetic field the function u may represent magnetization $\vec{M} = (M_X, M_Y, M_Z)^T$, f—magnetic flux density in measurement points $\vec{B} = (B_X, B_Y, B_Z)^T$, and g—multipole moments or magnetic flux density in locations other than measurement points (field extrapolation). In this case, the relation between the values of the field and the sources is given by the equation

$$\int_V K(x, y, z, x', y', z')\vec{M}(x', y', z')\, dV = \vec{B}(x, y, z), \tag{3}$$

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where K is an equation kernel that can be presented as a matrix function

$$K = \begin{pmatrix} K_{XX} & K_{XY} & K_{XZ} \\ K_{YX} & K_{YY} & K_{YZ} \\ K_{ZX} & K_{ZY} & K_{ZZ} \end{pmatrix},$$

x, y, z—coordinates of measurement point, $x',\ y',\ z'$—coordinates of volume point V.

Magnetic dipole moment $\vec{P} = (P_X, P_Y, P_Z)^T$ is calculated by the formula

$$\int_V \vec{M}(x',\ y',\ z')\,dV = \vec{P}. \tag{4}$$

In its turn, the function u may represent current source strength I for static electric field, f—the values of potential at measurement points φ, and g—multipole moments or values of the potential in other conditions than during the measurement. When the sources of current are located at the surface of object S, the values of the field are related to the sources by the equation

$$\int_S \Phi(x, y, z, x', y', z')\, I(x', y', z')\,dS = \varphi(x, y, z), \tag{5}$$

where Φ is an equation kernel.

Dipole electric moment is calculated as

$$\int_S I(x',\ y',\ z')\vec{r}\,dS = \vec{P}, \tag{6}$$

where $\vec{r} = (x',\ y',\ z')^T$.

This paper considers the class of problems with known source locations or known boundaries of source distribution but unknown strength. It is assumed that a priori information about the strength of field sources is not available.

Eqs. (3) and (5) are Fredholm equations of the 1st kind. The solution of these equations is an ill-posed problem.

The problem of magnetic field extrapolation based on equivalent surface sources is investigated in [1]. Directly, recovery of magnetic field sources, i.e. solution to Problem (1), is covered in [2–4]. The purpose of these papers is to develop algorithms for solving the inverse problem. No accuracy assessment for obtained solution to the inverse problem, or extrapolation error assessment for the general case is performed.

The purpose of this study is to estimate calculation error for the functionals of kind (2).

2. On the choice of solution method

In accordance with general notion regarding the solution of ill-posed problems, a regularization algorithm is formulated taking into account the information about the error of input data and operator. In case of Tikhonov method, a regularization parameter is selected based on residual method or generalized residual method and directly depends on the level of error. In iteration algorithms, the number of iterations depends on the error of input data. It has been theoretically proved that in general case it is impossible to obtain the solution to an incorrect problem if error of input data and operator are unknown [5,6]. However, in certain practical applications, a number of algorithms [7,8] that do not use error data might provide an acceptable solution.

Regularization error can be estimated for some classes of problems. In this case, a priori information is required for such estimation (for example, belonging of solution to some set) as well as the data on the error of input data and operator. In the considered statement of the problem, normal pseudosolution obtained on the basis of accurate input data is selected as an accurate solution for estimation of regularization error. However, the total error of problem solution cannot be found based on obtained estimations due to discrepancy between true (physical) solution and normal pseudosolution.

In classical theory of ill-posed problems, the obtained estimations of accuracy appear to be efficient at relatively low level of input data error but it is not always the case for solution of some practical problems.

When the field is measured in specially equipped laboratories, it is possible to obtain sufficiently accurate measurements and estimate the error of measurements. However, for engineering problems it may well be required to take measurements outside the laboratories when it is not feasible to move an object or it is too large in size. In such conditions, the external influence can be unavoidable and rather strong causing a large measurement error. When the actual error of input data is great, some methods of ill-posed problems solution directly relating the solution to measurement error (for example, residual method) are inefficient. In this case, it is difficult to estimate the accuracy of solution. Even if full-scale measurements are performed in well-controlled conditions (under moderate external influence), the error of measurement is governed by many factors and external effects and it is difficult to determine the actual error of measurement. In some cases, only upper error estimates can be obtained. However, if the upper estimates of measurement error are used for solution of inverse problem, it results in excessively smoothed solution and overestimation of obtained values in case of solution accuracy assessment.