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Efficient pricing of discrete arithmetic Asian options under mean reversion and jumps based on Fourier-cosine expansions

Chun-Sung Huang [a,e,∗], John G. O'Hara [b,d], Sure Mataramvura [c,e]

[a] Department of Finance & Tax, University of Cape Town, Rondebosch 7701, South Africa
[b] Centre for Computational Finance & Economic Agents, University of Essex, Colchester CO4 3SQ, UK
[c] Division of Actuarial Sciences, University of Cape Town, Rondebosch 7701, South Africa
[d] Mathematical Sciences, University of KwaZulu-Natal, Westville Campus, Durban 3001, South Africa
[e] African Collaboration for Quantitative Finance & Risk Research (ACQuFRR), University of Cape Town, Rondebosch 7701, South Africa

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ABSTRACT

We propose an efficient pricing method for arithmetic Asian options based on Fourier-cosine expansions. In particular, we allow for mean reversion and jumps in the underlying price dynamics. There is an extensive body of empirical evidence in the current literature that points to the existence and prominence of such anomalies in the prices of certain asset classes, such as commodities. Our efficient pricing method is derived for the discretely monitored versions of the European-style arithmetic Asian options. The analytical solutions obtained from our Fourier-cosine expansions are compared to the benchmark fast Fourier transform based pricing for the examination of its accuracy and computational efficiency.

1. Introduction

A topic of ongoing interest is the long standing hard problem of pricing arithmetic Asian options. The payoffs of these path-dependent exotics are based on the arithmetic average of the underlying prices monitored at fixed dates prior to maturity. The monitoring dates used to measure the arithmetic averages may also be taken at different frequencies, such as daily, weekly or monthly. Unlike its closely related geometric type, the prices of the more commonly traded arithmetic Asian options must be approximated numerically. This is mainly due to the absence of an analytically tractable solution for the distribution of the sum of log normally distributed random variables.

Asian options, introduced in 1987, are now widely traded in the commodities market as a hedging tool. For instance, various delivery companies in the gas market utilise Asian options to their advantage under risk management (see [1]). The popularity of Asian options arises mainly from its averaging effect, which is able to reduce possible risk of market manipulation in the price of the underlying at maturity. In addition, since averages move in a more stable way in comparison to individual prices, the volatility inherent in the underlying price is reduced as a result. Further information on Asian options with its history and evolution may also be found in [2,3].

It has been well documented that the prices of certain asset classes, such as commodities, show evidence of mean reversions and jumps. Hence, the pricing of options within these asset classes has also become an important focus in the

∗ Corresponding author at: Department of Finance & Tax, University of Cape Town, Rondebosch 7701, South Africa.
E-mail address: Chun-Sung.Huang@uct.ac.za (C.-S. Huang).

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field of quantitative finance. For example, due to the impact of relative prices on the supply of both copper and oil, prices tend to fluctuate randomly around some equilibrium level (see [4]). In addition to the above mentioned commodities, Bessembinder et al. [5] provide strong evidence supporting mean reversion in nine commodity markets, while Casassus and Collin-Dufresne [6] reveal the existence of such anomaly in the precious metals market. Apart from commodities, however, evident motivating the patterns of mean reversion has also been found in exchange rates and, interestingly, certain stock prices as well (see [7,8]).

In addition to the mean reverting property, jumps in the underlying price dynamic is another prominent feature. For instance, Jorion [9] examined the prices of stock market indices and exchange rates for discontinuities, while Geman and Roncoroni [10] and Seifert and Uhrig-Homburg [11] conducted investigations to provide empirical evidence of jumps in the power market. In addition, Geman and Roncoroni [10] found further evidence in support of mean reversion. Further empirical evidence in support of jumps in commodity prices may also be found in the current literature (see [12,13], and [14]).

Combining the fundamental ideas above, we propose an efficient pricing method for discrete arithmetic Asian options under a pricing dynamic which exhibits both jumps and mean reversion. Essentially, the model is a jump–diffusion extension of the one utilised by Fusai et al. [15] (as proposed by Chung and Wong [16]). Apart from accuracy in the pricing, computational efficiency is also of equal importance, if not more, particularly, for high frequency traders. Such notion brings about the non-trivial problem of finding a reasonable tradeoff between accuracy and efficiency in the pricing methods. As a result, efficient pricing methods of exotic options have also gained much interest from both practitioners and academics alike.

In option pricing, the valuation of complex contracts requires efficient numerical methods. The conditional expectation of the option payoff under the risk-neutral measure can be bridged with the solution of a partial differential equation through the well-known Feynman–Kac theorem. It then follows that various numerical pricing techniques, including numerical integration, can be developed. These numerical integration techniques rely on the transformation into the Fourier domain, which is particularly useful especially since the density function of many relevant underlying price process, required for the integration in the original domain, is not known. However, its Fourier transform, the characteristic function often is. It then follows that the fast Fourier transform (FFT) method, introduced by Carr and Madan [17] and Dempster and Hong [18], may be applied to calculate the option price efficiently. However, Fang and Oosterlee [19] proposed a novel pricing method, the Fourier-cosine expansions (COS method), as an alternative to the FFT. Such method could further improve the speed in the pricing.

In this paper, we propose to price discrete arithmetic Asian options under the assumption of mean reversion and jumps with the COS method. We show through numerical analysis that the COS method is indeed more efficient than the benchmark FFT, used by Chung and Wong [16]. It was also shown in [16] that the FFT is superior to the commonly implemented Monte Carlo simulation.

The remainder of this paper is organised as follows. In Section 2, we explore the proposed diffusion model for the underlying price dynamics with mean reversion and jumps. The joint characteristic function between the arithmetic average of the asset prices and its terminal value is also derived. Section 3 briefly introduced the COS method and the procedures to price the Asian option in question. We present a set of numerical results and analysis in Section 4 to evaluate the accuracy and efficiency of the COS method benchmarked to the FFT. The sensitivity of the COS prices to the underlying parameters is then analysed. We conclude the paper in Section 5.

2. Price process with mean reversion and jumps

2.1. Model specification

Let $(\Omega, \mathcal{F}, \mathbb{Q})$ be a probability space on which a Brownian motion process W_t and a Poisson process N_t, with intensity $\lambda > 0$, is defined for $0 \le t \le T$. Furthermore, we assume independence between the Brownian motion and Poisson process. Suppose $\mathbb{Q}$ is the risk neutral measure under which the price process is governed by the following dynamics:

$$dS_t = \kappa\left(\theta - \frac{\mu\lambda}{\kappa} - S_{t\_}\right)dt + \sigma\sqrt{S_{t\_}}dW_t + JdN_t, \tag{1}$$

where $J \sim Exp(\mu)$ and $N_t \sim Poi(\lambda t)$.

The model proposed here is an extension of the Fusai et al. [15] model, whereby a jump component has been added to the original spot price process, which is defined as a square root process driven by a Brownian motion. The jump size J and its arrival rate N_t are independent, and are modelled with an exponential distribution and a Poisson process, respectively. More specifically, the proposed price process is a CIR model with an exponential jump extension. Further justifications for the specific choice of jump dynamics can be found in [20,21], with the latter suggesting the non-existence of an analytical solution under lognormal jumps.

The use of the CIR as a base model gives rise to two main advantages in terms of Asian option pricing. Firstly, since we are interested in the average price of the underlying, the existence of the characteristic function for $\int_0^T r_t dt$ in the CIR model, used widely in the modelling of interest rates, helps simplify the problem at hand. Secondly, instead of a log price, by