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On the expected penalty functions in a discrete semi-Markov risk model with randomized dividends

Kam Chuen Yuen [a], Mi Chen [b,*], Kam Pui Wat [a]

[a] Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong

[b] School of Mathematics and Computer Science & FJKLMAA, Fujian Normal University, Fuzhou 350117, China

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ABSTRACT

This paper considers the expected penalty functions for a discrete semi-Markov risk model with randomized dividends. Under the model, individual claims are governed by a Markov chain with finite state space, and the insurer pays a dividend of 1 with a probability at the end of each period if the present surplus is greater than or equal to a threshold value. Recursive formulae and the initial values for the discounted free penalty functions are derived in the two-state model. A numerical example is provided to illustrate the impact of dividend payments on ruin probabilities.

1. Introduction

Survival probability in a semi-Markov risk model was first investigated by Janssen and Reinhard [1], in which the surplus process not only depends on the current state but also on the next state of an environmental Markov chain. Recently, Albrecher and Boxma [2] generalized the approach of Janssen and Reinhard [1] and studied the corresponding discounted penalty function by means of Laplace–Stieltjes transforms. Cheung and Landriault [3] further investigated the problem of Albrecher and Boxma [2] by relaxing some assumptions pertaining to the interclaim time distribution.

For the discrete-time semi-Markov risk model with a restriction imposed on the total claim size, Reinhard and Snoussi [4,5] derived recursive formulae for the distribution of the surplus just prior to ruin and that of the deficit at ruin in a special case. Chen et al. [6,7] relaxed the restriction of Reinhard and Snoussi [4,5] and derived recursive formulae for computing the expected discounted dividends and survival probabilities for the model. As was mentioned in [7], the discrete-time semi-Markov risk model without restriction embraces some existing discrete-time risk models including the compound binomial model (with time-correlated claims) and the compound Markov binomial model (with time-correlated claims) which have been extensively studied by various authors; see, for example, Cossette et al. [8,9], Yuen and Guo [10], Xiao and Guo [11] and references therein. This motivates us to carry out further ruin analysis for the discrete-time semi-Markov risk model.

The randomized dividend strategy was studied by Tan and Yang [12], Bao [13], Landriault [14], He and Yang [15], and Yuen et al. [16], for the compound binomial model. Under this dividend payment strategy, the insurer pays a dividend of 1 with probability $1-\alpha$ when the surplus is greater than or equal to an arbitrary given non-negative integer x. In this paper, we incorporate randomized dividends into the discrete-time semi-Markov risk model of Chen et al. [6,7], and examine the corresponding discounted free Gerber–Shiu penalty function.

The rest of the paper is organized as follows. In Section 2, we present the mathematical formulation of the discrete semi-Markov model with randomized dividends. In Section 3, we derive recursive formulae for computing the discounted

* Corresponding author.
E-mail address: chenmi0610@163.com (M. Chen).

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free Gerber–Shiu penalty function for the model. In Section 4, we obtain two important equations for determining the required initial values when applying the recursive formulae. Sections 5 and 6 are devoted to finding the initial values for the case with $x = 0$. Finally, a numerical example is given in Section 7.

2. The risk model

Let $(J_n, n \in \mathbb{N})$ be a homogeneous, irreducible and aperiodic Markov chain with finite state space $M = \{1, \dots, m\}$ $(1 \le m < \infty)$. Its one-step transition probability matrix is given by

$$\mathbf{P} = (p_{ij})_{i,j \in M}, \quad p_{ij} = \mathbb{P}(J_n = j | J_{n-1} = i,\ J_k, k \le n-1),$$

with a unique stationary distribution $\boldsymbol{\pi} = (\pi_1, \dots, \pi_m)$. The insurer's surplus (without paying dividends) at the end of the tth period, X_t, has the form

$$X_t = u + t - \sum_{i=1}^{t} Y_i, \quad t \in \mathbb{N}_+, \tag{1}$$

where $u \in \mathbb{N}$ is the initial surplus and Y_i denotes the total amount of claims in the ith period. We further assume that a premium of 1 is received at the beginning of each time period, and that Y_t's are nonnegative integer-valued random variables. The distribution of Y_t's is governed by the environmental Markov chain $(J_n, n \in \mathbb{N})$ in the way that (J_t, Y_t) depends on $\{J_k, Y_k;\ k \le t-1\}$ only through J_{t-1}. Define

$$g_{ij}(l) = \mathbb{P}(Y_t = l,\ J_t = j | J_{t-1} = i,\ J_k, Y_k, k \le t-1), \quad l \in \mathbb{N},$$

which describes the conditional joint distribution of Y_t and J_t given the previous state J_{t-1} and plays a key role in the following derivations. Note that $p_{ij} = \sum_{l=0}^{\infty} g_{ij}(l)$. We refer the readers to Reinhard and Snoussi [4,5] for more details about the model.

We now modify the surplus process (1) by allowing dividend payments. Specifically, we assume that the insurer will pay a dividend of 1 with probability $1-\alpha$ at the end of each period if the present surplus is greater than or equal to a threshold value $x \in \mathbb{N}$. Then the modified surplus at the end of the tth period is given by

$$U_t = u + t - \sum_{i=1}^{t} Y_i - \sum_{i=1}^{t} \gamma_i \mathbf{1}_{(U_{i-1} \ge x)}, \quad t \in \mathbb{N}_+, \tag{2}$$

where $\mathbf{1}_A$ is the indicator function of event A and γ_i is a series of i.i.d. random variables that are independent of Y_i with $\mathbb{P}(\gamma_i = 0) = \alpha > 0$ and $\mathbb{P}(\gamma_i = 1) = 1 - \alpha$.

Let $\tau = \inf\{t \in \mathbb{N}_+ : U_t < 0\}$ be the time of ruin. The Gerber–Shiu expected discounted penalty function given the initial surplus u and the initial environment state i is defined as

$$m_i(u) = E(v^{\tau} \omega(U_{\tau-}, |U_\tau|) \mathbf{1}_{(\tau < \infty)} | U_0 = u, J_0 = i), \quad i \in M,\ u \in \mathbb{N}, \tag{3}$$

where $\omega(x, y)$ is a nonnegative bounded function and $0 < v \le 1$ is the discounted factor. If $v = 1$ and $\omega(x, y) \equiv 1$, then $m_i(u)$ becomes

$$\psi_i(u) = \mathbb{P}(\tau < \infty | U_0 = u, J_0 = i), \quad i \in M,\ u \in \mathbb{N},$$

which is the ultimate ruin probability given the initial surplus u and the initial environment state i. Let $\phi_i(u) = 1 - \psi_i(u)$ be the corresponding survival probability.

For all i and j, we assume that

$$\mu_{ij} = \sum_{k=0}^{\infty} k g_{ij}(k) < \infty,$$

and define

$$\mu_i = \sum_{j=1}^{m} \mu_{ij}, \quad i \in M.$$

Here we consider the following positive safety loading condition for the model

$$\sum_{i=1}^{m} \pi_i \mu_i < 1 - (1 - \alpha) = \alpha,$$

which ensures that ruin is not certain.

In this paper, we only consider the case with $v = 1$ and $m = 2$. Our aim is to derive a recursive formula for computing $m_i(u)$.