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A numerical approach for solving Volterra type functional integral equations with variable bounds and mixed delays

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ABSTRACT

In this paper, the Taylor collocation method has been used the integro functional equation with variable bounds. This method is essentially based on the truncated Taylor series and its matrix representations with collocation points. We have introduced the method to solve the functional integral equations with variable bounds. We have also improved error analysis for this method by using the residual function to estimate the absolute errors. To illustrate the pertinent features of the method numeric examples are presented and results are compared with the other methods. All numerical computations have been performed on the computer algebraic system Maple 15.

1. Introduction

Integral equations play very significant role linear and nonlinear functional analysis and their applications [1]. They are mostly in connection with functional equations. Functional equations occur with difference, differential and integral forms [2]. (FDE) have been studied in several papers [2–12]. Functional integral equations (FDEs) and their systems have a major importance in modeling many phenomena such as biology, ecology, physics and engineering so they have been studied in several papers [13–18]. An integro functional equation is illustrated by

$$F\left\{x, \varphi(x), \varphi[f(x)], \int_{x_0}^{x} K_r(x,t,\varphi(t), \varphi[f(t)])\,dt\right\} = 0.$$

FDEs are usually difficult to solve analytically; so there are particular methods that have solved them numerically. Up to now to obtain numerical solutions of the first and second kind of functional integral and integro-differential equations have been used an expansion method based on Chebyshev interpolation [7,8], Lagrange collocation method [16], Chebyshev collocation method [9,10], variational iteration method (VIM) [6] and Legendre collocation method [11].

In this article we want to find truncated Taylor series solution of integro functional equation with variable bounds represented by

$$\sum_{k=0}^{m_1} P_k(x)\, y(\alpha_k x + \beta_k) = f(x) + \sum_{r=0}^{m_2} \lambda_r \int_{u_r(x)}^{v_r(x)} K_r(x,t)\, y(\mu_r t + \gamma_r)\,dt \qquad (1)$$

where $P_k(x)$, $f(x)$, $K_r(x,t)$, $u_r(x)$, $v_r(x)$ are continuous functions on the interval $[a,b]$, $a \le u_r(x) \le v_r(x) \le b$ and $\alpha_k, \beta_k, \lambda_k, \mu_k, \gamma_k$ are appropriate constants.

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The main purpose of this study is to solve (1) using the Taylor matrix method. Since the beginning of 1994, Taylor, Chebyshev, Legendre, Laguerre, Hermite and Bessel collocation and matrix methods have been used by Sezer et al. [19–28] to solve differential, difference, integral, integro-differential, delay differential equations and their systems. In this article, by modifying and developing matrix and collocation methods studied in [19,24,25], we will find the approximate solutions of the system (1) in the truncated Taylor series form

$$y(x) \cong y_N(x) = \sum_{n=0}^{N} y_n x^n, \quad y_n = \frac{y^{(n)}(0)}{n!} \tag{2}$$

where y_n, $(n = 0, 1, \ldots, N)$ are unknown coefficients to be determined.

2. Fundamental relations

To find the approximate solution of (1) in the form of (2) first we convert the solution defined by (2) for $n = 0, 1, 2, \ldots, N$ to the following matrix form:

$$\mathbf{y}(x) = \mathbf{X}(x)\,\mathbf{Y} \tag{3}$$

where

$$\mathbf{X}(x) = \begin{bmatrix} 1 & x & x^2 & \cdots & x^N \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} y_0 & y_1 & y_2 & \cdots & y_N \end{bmatrix}^T.$$

By putting $x \to \alpha_k x + \beta_k$ in the relation (3) we obtain the matrix form

$$\mathbf{y}(\alpha_k x + \beta_k) \cong \mathbf{y}_N(\alpha_k x + \beta_k) = \mathbf{X}(\alpha_k x + \beta_k)\mathbf{Y}$$

where

$$\mathbf{X}(\alpha_k x + \beta_k) = \begin{bmatrix} (\alpha_k x + \beta_k)^0 & (\alpha_k x + \beta_k)^1 & (\alpha_k x + \beta_k)^2 & \cdots & (\alpha_k x + \beta_k)^N \end{bmatrix}_{1\times(N+1)}.$$

From the binomial expansion of the $(\alpha_k x + \beta_k)^N$, we can write the relation between the matrices $\mathbf{X}(\alpha_k x + \beta_k)$ and $\mathbf{X}(x)$ is

$$\mathbf{X}(\alpha_k x + \beta_k) = \mathbf{X}(x)\,\mathbf{B}(\alpha_k, \beta_k) \tag{4}$$

where

$$\mathbf{B}(\alpha_k, \beta_k) = \begin{bmatrix} \binom{0}{0}\alpha_k^0\beta_k^0 & \binom{1}{0}\alpha_k^0\beta_k^1 & \binom{2}{0}\alpha_k^0\beta_k^2 & \cdots & \binom{N}{0}\alpha_k^0\beta_k^N \\ 0 & \binom{1}{1}\alpha_k^1\beta_k^0 & \binom{2}{1}\alpha_k^1\beta_k^1 & \cdots & \binom{N}{1}\alpha_k^1\beta_k^{N-1} \\ 0 & 0 & \binom{2}{2}\alpha_k^2\beta_k^0 & \cdots & \binom{N}{2}\alpha_k^2\beta_k^{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \binom{N}{N}\alpha_k^N\beta_k^0 \end{bmatrix}_{(N+1)\times(N+1)}.$$

By substituting the relation (4) into the relation (3), we reach the matrix relation

$$\mathbf{y}(\alpha_k x + \beta_k) \cong \mathbf{y}_N(\alpha_k x + \beta_k) = \mathbf{X}(x)\mathbf{B}(\alpha_k, \beta_k)\mathbf{Y}. \tag{5}$$

Similarly, it is clear that the matrix form of $\mathbf{y}(\mu_r t + \gamma_r)$ is

$$\mathbf{y}(\mu_r t + \gamma_r) \cong \mathbf{y}_N(\mu_r t + \gamma_r) = \mathbf{X}(t)\mathbf{B}(\mu_k, \gamma_k)\mathbf{Y}. \tag{6}$$

Now, we convert the kernel functions $K_r(x, t)$ to the matrix forms, by means of the following procedure.

The function $K_r(x, t)$ can be expressed by the truncated Taylor series as

$$K_r(x, t) = \sum_{p=0}^{N}\sum_{q=0}^{N} k_{p,q}^r x^p t^q \tag{7}$$

where

$$k_{p,q}^r = \frac{1}{p!q!}\frac{\partial^{p+q}K_r(0, 0)}{\partial x^p \partial t^q}, \quad p, q = 0, 1, \ldots, N, \ r = 0, 1, \ldots, m_2.$$

The expressions (7) can be written in the matrix forms

$$\mathbf{K}_r(x, t) = \mathbf{X}(x)\mathbf{K}_r\mathbf{X}^T(t), \qquad \mathbf{K}_r = \left[k_{p,q}^r\right], \quad p, q = 0, 1, \ldots, N, \ r = 0, 1, \ldots, m_2 \tag{8}$$

where $\mathbf{K}_r = \left[k_{p,q}^r\right]$, $p, q = 0, 1, \ldots, N$, are the Taylor coefficients matrices of functions $\mathbf{K}_r(x, t)$ at the point $(0, 0)$.