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Reliability nonparametric Bayesian estimation for the masked data of parallel systems in step-stress accelerated life tests

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ABSTRACT

The accelerated life tests with two groups of step-stress levels are considered for the parallel systems, in which the masked data are observed. We assume the power function as accelerated function for the life transformation between different stress levels, which covers the shortage of linear accelerated function. Then the estimators of coefficient vectors in accelerated function are obtained. The relationship among survival and subsurvival functions of components is discussed under a regular condition. It does not require that the discontinuity points of survival functions have to be disjointed. With the help of that transformational relationship, nonparametric Bayesian estimators of reliability functions corresponding to any components' set are derived. Due to the complexity of masked data, a group of computational algorithms are developed to obtain the estimates. Finally, a simulated example is presented to illustrate the proposed nonparametric Bayesian method.

1. Introduction

In the field of reliability analysis, the accelerated life test (ALT) is commonly used to shorten the test time and reduce the economic cost for high reliability products. Usually, the test units are subjected to higher than usual levels of stress. There are three basic types of ALT models which consist of constant stress accelerated life test (CSALT), step-stress accelerated life test (SSALT), and progressive stress accelerated life test (PSALT).

Many researchers have studied the Bayesian method for product lifetimes in ALT. Xu et al. [1] studied the Bayesian analysis of constant-stress accelerated life test for the Weibull distribution products. Fan et al. [2] discussed the Bayesian reliability inference of a series system under the step-stress accelerated life test while the components are assumed to have Weibull lifetime distribution. Abdel-Hamid and Abushal [3] made a statistical inference on progressive-stress model for the exponentiated exponential distribution under type-II progressive hybrid censoring. The statistical inference of competing risks model in the CSALT and SSALT with progressive hybrid censoring were discussed by Wu et al. [4] and Zhang et al. [5] respectively. The basic assumption of these works is that the life distribution of items under every accelerated stress level is known, such as exponential or Weibull distribution. Although these specific parametric lifetime distributions are appealing from a statistical point of view, Hu et al. [6] pointed out that the assumption with known distribution may be unreasonable in many practical applications. For instance, the underlying life distribution of some new products is unknown. If the assumed distribution does not provide a good approximation of the actual failure mechanism, significant errors in the extrapolation

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of ALT will occur. Thus the nonparametric model in ALT is of great interest which does not require specific distributional assumptions.

A few researchers have discussed the nonparametric approach for the failure lifetime data under SSALT. Shaked and Singpurwalla [7] early considered the inference for SSALT by the nonparametric approach. In their paper, a collection of stress patterns no less than two are introduced to make the estimation. Tyoskin and Krivolapov [8] suggested the nonparametric method under SSALT for obtaining the hypothesis test, estimation of time-transformation function and lower confidence bounds of reliability measures. Bagdonavicius and Nikulin [9] estimated the nonparametric reliability of items from two steps SSALT. They also discussed the asymptotic properties of the estimators. Recently, Hu et al. [6] provided the nonparametric estimation procedure for obtaining upper confidence bounds of the cumulative failure probability and the goodness of fit test. In their article, a proportional hazards model was proposed for the SSALT. So far only a few researchers use nonparametric Bayesian method to analyze the SSALT. Salinas-Torres et al. [10] reported a nonparametric Bayesian estimator with respect to the series system based on multiple competing risks, some results of which are corrected by Polpo and Sinha [11]. Xu and Tang [12] provided the nonparametric Bayesian analysis for the masked data of series system. Furthermore, Polpo et al. [13,14] developed the nonparametric Bayesian results for parallel systems and hybrid coherent systems. Nevertheless, these researchers studied the nonparametric Bayesian inference under usual stress condition and did not involve the ALT. Masked competing-failure data often happens in ALT in that the specific failure cause is not observed. For instance, the documentation needed for cause type identification is lost, or the cause type is difficult to determine, or the cause type detection is expensive to do for each subject, etc. It is of great significance to present the nonparametric Bayesian reliability for the masked data under ALT.

In this paper, we consider the reliability estimation of parallel systems in SSALT with two groups of step-stress levels. In Section 2, the SSALT model and some basic assumptions are elaborated. Different from the usual linear accelerated function, we adopt the power function as the general accelerated function, which may be more suitable for the nonlinear case. The estimators of the coefficients in accelerated function (call them accelerated coefficients) are also obtained by the nonparametric method. Meanwhile, the coefficient vectors connecting the accelerated coefficient and stress levels are estimated by the least square method. The interrelations among survival, subsurvival functions of components are studied for the parallel system in Section 3. The restriction that there are no common discontinuity points between components' survival functions is replaced by a relaxed condition. To obtain the reliabilities of parallel systems as well as their components, nonparametric Bayesian method is presented in Section 4 for the masked data in SSALT. As a result, the posterior estimation of reliability function is obtained by a computational algorithm. In Section 5, an example of numerical simulation is showed to illustrate the feasibility and effect of our proposed method. Section 6 summarizes the results and suggests some future possible research.

2. Step-stress ALT model and coefficient vectors estimation

2.1. Model description and assumptions

In industrial step-stress life testing experiments it is common practice to put several items on the test under various stress patterns [7]. In order to determine the coefficients of accelerated function without any distributional assumption, a SSALT with two groups of step-stress levels is carried out. Assume k stress levels $S_1 < S_2 < \cdots < S_k$ and $k-1$ transition times $\tau_1 < \tau_2 < \cdots < \tau_{k-1}$. Two groups of parallel systems with J independent components, $j = 1, 2, \ldots, J$, are tested simultaneously in the following manner. Initially, the first group of N_1 items begin the test at stress level S_1. At time τ_1, all of the unfailed systems are moved to the stress level S_2 until time τ_2. Repeat the process until at stress level S_{k-1} and time τ_{k-1} and all remaining survivors are removed. At the beginning time of the first group, the second group of N_2 systems is placed simultaneously at stress level S_2 until time τ_1. Then the survivors are maintained under a higher stress level S_3 up to τ_2. Similarly, the process is repeated until time τ_{k-1}. At the moment τ_{k-1}, the test is terminated and all remaining survivors are removed. For notational simplicity, we identify the two groups of stress patterns by $\gamma = 1, 2$. Schematically, the test can be represented as in Fig. 1.

For two groups of step-stress patterns $\gamma = 1, 2$, let $\tau_0 = 0$ and denote the number of failed systems in $(\tau_{i-1}, \tau_i]$ as $n_i^{(\gamma)}$ under the step-stress pattern γ for $i = 1, 2, \ldots, k-1$. Then the number of censored systems under stress pattern γ is $C_\gamma = N_\gamma - \sum_{i=1}^{k-1} n_i^{(\gamma)}$. The observed data between $(\tau_{i-1}, \tau_i]$ are recorded as $\left\{t_{i,l}^{(\gamma)}, M_{i,l}^{(\gamma)}\right\}$, $\gamma = 1, 2,\ i = 1, 2, \ldots, k-1,\ l = 1, 2, \ldots, n_i^{(\gamma)}$, where $t_{i,l}^{(\gamma)}$ is the failure time of the lth failed system between $(\tau_{i-1}, \tau_i]$ under stress pattern γ, and $M_{i,l}^{(\gamma)}$ is a set of components corresponding to $t_{i,l}^{(\gamma)}$ that identify the true cause of failure.

Under the stress level $S_i,\ i = 1, 2, \ldots, k$, we define the distribution function of component j as $F_{ij}(t) = \Pr\left(T_{ij} \le t \,|S_i\right)$ and the subdistribution function as $F_{ij}^*(t) = \Pr\left(T_i \le t, \delta_i = j \,|S_i\right)$ for $j = 1, 2, \ldots, J$, where T_i is the system lifetime under stress level S_i, δ_i is the real failure cause of system at T_i. The corresponding survival and subsurvival functions are defined as $R_{ij}(t) = 1 - F_{ij}(t) = \Pr\left(T_{ij} > t \,|S_i\right)$ and $R_{ij}^*(t) = 1 - F_{ij}^*(t) = \Pr\left(T_i > t, \delta_i = j \,|S_i\right)$. Polpo et al. [13] defined the distribution and subdistribution functions for parallel systems with respect to the risk subset Δ, a nonempty subset of $\{1, 2, \ldots, J\}$. Under the stress level S_i, we generalize the distribution and subdistribution functions with subset Δ as $F_{i\Delta}(t) = \Pr\left(\max_{j\in\Delta} T_{ij} \le t \,|S_i\right)$ and $F_{i\Delta}^*(t) = \Pr\left(T_i \le t, \delta_i \in \Delta \,|S_i\right)$. Similarly, the corresponding survival and subsurvival