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Default prediction with the Merton-type structural model based on the NIG Lévy process

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ABSTRACT

Merton's model (Merton, 1974) has long been a standard for estimating company's probability of default (PD) for listed companies. The major advantage of Merton's model is the use of current market prices to determine the probability of default. The logic behind the model is simple; the market prices best reflect all the relevant information (being forward looking estimates of company's prospect) and should be (and are) superior to the balance sheet disclosures, which at best are ex post realisations of company's performance. It is thus a pity that the benefits (strengths) of Merton's model are hindered by a significant shortcoming of the model namely the assumption of normally distributed returns.

As numerous authors point out (Barndorff-Nielsen, 1997 [5,6]; Prause, 1999; Eberlein, 2001; Brambilla et al. 2015), stock returns are not normally distributed which significantly limits the use of model in practice. Moreover the estimates of PDs can be biased downwards exposing the banks to the possibility of undercapitalisation and systematic shocks.

It is the purpose of this paper to remedy this situation. Firstly we extend the Merton model by allowing for normal inverse Gaussian (NIG) distributed returns. As several authors point out using the examples of options (Schoutens, 2009), NIG in most cases provides a robust statistical platform for estimating stock returns. We further extend our approach by constructing a robust EM algorithm for estimating PDs within the Merton NIG framework.

We also test the reliability of the NIG improved Merton model against classical Merton's model for estimating PDs. Applying our results to Ljubljana stock exchange we find that the PD estimates using classical Merton's model are biased, whereas the estimates from NIG Merton's model are robust.

1. Introduction

This paper discusses two structural models for PD estimation. The first is the classical Merton model, first developed by [1] and termed by [2] as a first-generation structural model. The second is the Merton NIG model, developed in this paper. The Merton NIG model is derived from the classical Merton model, but market values of assets follow a different stochastic process: normal inverse Gaussian Lévy process instead of Brownian motion, hence the inclusion of the NIG term. The term *structural* relates to the model assumptions, in which a firm's estimated PD is a function of its capital *structure* as well as the

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variability and trend in asset values over time. Structural models are relevant to PD estimation as they are based on the share price movements of stock exchange listed firms observed daily, which makes it possible to estimate the PD of such firms at any point in time and over any time horizon. For the same reason, however, the applicability of such models is limited to listed firms. The credit portfolios of Slovenian banks consist primarily of unlisted rather than listed firms. Structural models are thus inapplicable to the former group, or at least do not have the same temporal flexibility. Such models are used or marketed by major international banks and credit ratings institutions, viz. e.g. Moody's KMV Expected Default Frequency model [3] and the Fitch Equity Implied Rating and Probability of Default Model [4].

The classical Merton model assumes a simple capital structure and normally distributed returns to the firm's assets. Although the capital structure in this model is simplistic and typically does not reflect reality, the implications of this assumption for the biasedness of the PD estimates will not be discussed in detail in this paper. Here we will focus on the assumption that the natural logarithm of returns on firms' assets is normally distributed, and the effect of this assumption on PD estimates within structural models. Several authors [5–9] have found that log returns on share values, on the basis of which PD is estimated, are not adequately characterised by a normal distribution as implied by Brownian motion. If returns are not normally distributed, the assumptions of the classical Merton model are breached, and the resulting PD estimates may be biased.

The distribution of actual returns is often better described by a model in which asset values evolve according to a Lévy process in which log returns have a normal inverse Gaussian distribution. The NIG Lévy process, as this is called, has several convenient mathematical properties and its empirical adequacy in describing the movement of share values has been confirmed by numerous studies. It is therefore used in this paper for the purposes of estimating PD. The literature on the application of the NIG Lévy process to estimating PD is not extensive. [10] makes the important finding that the difference in PD estimates between the classical and the generalised hyperbolic models is negligible when the time remaining to maturity $T - t$ is a year or more, but does not provide a wider empirical study of these differences. [11] performs a one-factor copula on the basis of the NIG distribution for estimating conditional PD, with previously estimated unconditional PD entering the model. [12], for example, uses variants of Lévy processes for estimating credit derivatives, but explicitly do not deal with methods of PD estimation. [9] compares PD estimates based on two Lévy processes: variance gamma (VG) and NIG. Their motivation is similar to that of this paper, namely to overcome the potentially erroneous assumption of the normally distributed log returns on firm assets, in PD estimation. However, they do not compare PD estimates from the classical Merton model and a VG/NIG process. This paper attempts to help fill the gap in empirical studies in this area. To this end we develop a structural model for estimating firms' PD that is based on a NIG Lévy process.

This paper elaborates procedures for estimating a firm's PD. For both the classical Merton and Merton NIG models, *Expectation–Maximisation* (EM) algorithms are devised on the lines of [13–15], in which the parameters of the relevant distributions are estimated by the *method of maximum likelihood* (ML). ML parameter estimates have attractive theoretical properties [16]. Large-sample theory shows that, under reasonable conditions, ML sample estimates of parameters are consistent, unbiased and asymptotically normally distributed as the sample size tends to infinity. These theoretical properties permit valid inference, which is a desirable property in PD estimation. A comparison is made of the PD estimates from the classical Merton model and the Merton NIG model on empirical data.

This paper is organised as follows. Sections 2–3 set out the assumptions of the two structural models. Section 4 sets out the EM algorithm as a way of estimating PDs. Section 5 applies the two structural models to empirical data for particular Slovenian firms listed on the Ljubljana Stock Exchange in 2004 and 2005. Section 6 concludes the findings of this paper.

2. Assumptions of the classical Merton [1] model

In the Merton model, equity represents a call option on the part of equity holders on firm assets. The firm's financing is simple. It consists of one class of equity issued at time t (shares E_t) and debt issued at time t (zero-coupon bonds D_t). D_t matures at T with a value of L, which is the exercise price of the call option. L includes accrued interest reflecting the firm's risk. Debt holders finance the firm with D_t at time t and receive an amount equal to $\min[A_T, L]$ at time T, where A_T is the market value of the assets at maturity. Under this financing structure the firms asset value at t is given by

$$A_t = E_t + D_t. \tag{1}$$

The asset value A_t is governed by geometric Brownian motion

$$d\ln A_t = \mu_A dt + \sigma_A dZ_t, \tag{2}$$

where μ_A is the constant return on the firm's assets, σ_A is the constant standard deviation of this return and Z_t is a standard normal random variable. Using Itô's lemma, the solution to (2) is

$$A_T = A_t e^{Z_{T-t}}, \tag{3}$$

where $(T - t)$ is the time remaining to maturity. From (3) it follows that the natural logarithm of the firms asset return, $\ln(A_T/A_t)$, is normally distributed

$$N \sim \left(\left(\mu_A - \frac{\sigma_A^2}{2} \right)(T - t), \sigma_A^2 (T - t) \right). \tag{4}$$