Journal of Computational and Applied Mathematics 311 (2017) 551–564

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Image inpainting using reproducing kernel Hilbert space and Heaviside functions

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ARTICLE INFO

Article history:
Received 17 July 2015
Received in revised form 25 March 2016

Keywords:
Image inpainting
Reproducing kernel Hilbert space
Heaviside function
Edge

ABSTRACT

Image inpainting, a technique of repairing damaged images, is an important topic in image processing. In this paper, we solve the problem from an intensity function estimation perspective. We assume the underlying image is defined on a continuous domain and belongs to a space spanned by a basis of a reproducing kernel Hilbert space and some variations of the Heaviside function. The reproducing kernel Hilbert space is used to model the smooth component of the image while Heaviside function variations are used to model the edges. The coefficients of the redundant basis are computed by the discrete intensity at undamaged domain. We test the proposed model through various images. Numerical experiments show the effectiveness of the proposed method, especially in recovering edges.

1. Introduction

Image inpainting is a technique of repairing corrupted/missing image intensity in a visually plausible way [1–6]. The concept of digital image inpainting was first introduced in [7]. Image inpainting has gained a wide range of applications. Examples include restoration of ancient frescoes [8], removing texts and scratches, correcting red eye effects in photos and reducing artifacts [9]. Existing image inpainting methods can be classified into several categories: variational/partial differential equation (PDE) based [7,10–12]; sparsity based [13,14]; texture synthesis based [15]; exemplar based [16,17] and hybrid approaches [18,19]. Note that, these methods are not completely independent. For example, some sparsity based methods may belong to the category of exemplar based methods.

Variational/PDE based approaches inpaint images by diffusing local image structure from known regions into unknown inpainting regions. In [7], the authors smoothly propagated information from the surrounding areas in the direction of isophote lines into unknown regions. Subsequently, many variational models have been applied to image inpainting. For example, in [2], the authors proposed the curve driven diffusion (CDD) method and in [10], the authors introduced a variational framework to restore images by minimizing total variation (TV). An Euler's elastica energy based model was then discussed in [20] and this model can connect level lines over larger distances than TV based methods. In [11], the authors proposed two models which combine TV and wavelet. In [21], a joint interpolation of the image gray-levels and isophote directions was proposed for automatic inpainting in a variational framework. In [22], the authors used Cahn–Hilliard

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http://dx.doi.org/10.1016/j.cam.2016.08.032
0377-0427/

equation to inpaint binary images. The authors then generalized it for grayvalue images by using subgradients of TV functional within the flow and discussed the stationary state of the proposed model in [23]. The above methods need to solve PDEs and are sometimes time consuming. An operator splitting method [24] was then proposed to speed up TV inpainting algorithm. All these PDE based methods take advantage of the property that neighbored pixels are correlative and use local information around the inpainting domain. These algorithms perform well for non-texture images with relatively smaller missing regions.

Texture synthesis based methods are popular for images with textures. The idea is to synthesize new image pixels by learning from similar regions in a texture sample [15,25]. In [15], the authors presented an algorithm to synthesize new texture by taking patches of existing texture and stitching them together. However, they perform not that well when handling natural images that have complex interactions between structure and texture boundaries.

For images containing both structures and textures, hybrid methods are sometimes used. In [18] for instance, texture synthesis method and variational/PDE are combined. It decomposes an image into structure and texture parts and reconstructs the structure part by using a PDE based algorithm and the texture part through a texture synthesis algorithm. It is still hard to reconstruct large structure regions.

In order to reconstruct large missing/corrupted regions, some exemplar based methods are proposed. Exemplar based inpainting methods combine texture synthesis method and isophote driven inpainting method. An exemplar-based inpainting method by using a priority value which determines the fill order for each patch was introduced in [16]. This technique can propagate both linear structure and texture into inpainting domain but has some difficulties in handling curved structures. In [26], the authors proposed a novel method by using the sparsity at the patch level for modeling patch priority and patch representation. Many other exemplar-based methods are introduced in [27–30].

Sparsity based inpainting methods assume the underlying image (or the patch) is sparse under a given transform, such as discrete cosine transform, or wavelet. For instance, the authors used sparse representation over a redundant dictionary technique to fill the hole patch-wisely in [14]. This kind of methods may introduce smooth effect when filling large missing regions.

Some of above methods focus on maintaining structure of the inpainting areas and some concern about texture synthesis. In this paper, we focus on maintaining structure of images. Different from the above mentioned methods, the proposed inpainting approach is based on a reproducing kernel Hilbert space (RKHS) and its extensions. RKHS has been used as a powerful tool in machine learning, but not much in image processing [31–33]. So far, RKHS has only been explored in denoising [34], colorization [35] and segmentation [36]. In [34], the authors proposed an adaptive kernel method to deal with image denoising problems. They treat additive noise in a unified framework using RKHS. It can preserve sharp edges. However, this method needs expensive computation. Inspired by the extensions of RKHS in machine learning, the authors in [35] used a well-known least square regression in RKHS for colorization problems. So far, we have not found RKHS based method for image inpainting. In this paper, we assume that the underlying image is defined on a continuous domain and belongs to a space spanned by a basis of a RKHS and variations of Heaviside function. The coefficients of the redundant basis are computed by the discrete intensity at undamaged domain. Our experiments show the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, we briefly review RKHS and its applications in one-dimension (1-D) signal and two-dimension (2-D) image smoothing. In Section 3, we present a new model for image inpainting problem and discuss the numerical algorithm. In Section 4, some experimental results are presented. Finally, we conclude this work in Section 5.

2. Review on RKHS

We will use splines based RKHS to model the continuous component of images, so we review RKHS, 1D and 2D splines as well as their applications in image smoothing.

2.1. Reproducing Kernel Hilbert space

Definition 1 (*Gram Matrix*)**.** Let χ be a set. Given a function $\kappa : \chi \times \chi \to \mathbb{R}$ and $x_1, \ldots, x_N \in \chi$, the matrix $K = (K_{i,j})^N$ with elements $K_{i,j} = \kappa(x_i, x_j)$, for $i, j = 1, \ldots, N$, is called the Gram matrix (or kernel matrix) of κ with respect to $x_1, \ldots, x_N$.

Definition 2 (*Positive Semidefinite Kernel*)**.** Let χ be a nonempty set. A function $\kappa : \chi \times \chi \to \mathbb{R}$ is called a positive semidefinite kernel, if for all $N \in \mathbb{N}$ and all $x_1, \ldots, x_N \in \chi$, Gram matrix K is semidefinite.

Given a subset $\chi \subset \mathbb{R}$ and a probability measure P on χ, we consider a Hilbert space with a family of real value functions $f : \chi \to \mathbb{R}$, with $\|f\|_{L^2(P)} < \infty$, and a corresponding inner product $\langle\cdot,\cdot\rangle_{\mathcal{H}}$ under which $\mathcal{H}$ is complete. The space $\mathcal{H}$ is a reproducing kernel Hilbert space (RKHS), if there exists a kernel $\kappa : \chi \times \chi \to \mathbb{R}$ with the following two properties: (a) for each $x \in \chi$, $\kappa(x,\cdot)$ belongs to the Hilbert space $\mathcal{H}$, and (b) κ has the so called reproducing property, i.e. $f(x) = \langle f, \kappa(x,\cdot)\rangle$, for all $f \in \mathcal{H}$, in particular $\kappa(x, y) = \langle\kappa(x,\cdot), \kappa(y,\cdot)\rangle_{\mathcal{H}}$. Any such kernel function must be positive semidefinite.

Wahba proposed two splines based RKHS for smoothing problems in [37]. Our proposed model is partially motivated from it. In what follows, we review 1-D and 2-D spline based RKHS [34,37,38].