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The hole-filling method and multiscale algorithm for the heat transfer performance of periodic porous materials

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HIGHLIGHTS

- A multiscale analysis and computation based on hole-filling method is proposed.
- Transient heat transfer problem of periodic porous materials is considered.
- Proof of the limiting process as the parameters of the weak phase go to zero is derived.
- Some numerical results are given in detail to validate the multiscale method developed.

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ABSTRACT

In this paper, we consider the transient heat transfer problem with rapidly oscillating coefficients in periodic porous materials. The hole-filling method through filling all holes with a very compliant material is investigated, and as the thermal conductivity parameters of the weak phase go to zero, the proof of the limiting process is derived in detail. Then, the multiscale analysis and computations based on the hole-filling method for the associated multiphase problem in a domain without holes are proposed. Finally, some numerical results are given, which show a good agreement and verify the feasibility of the hole-filling method.

1. Introduction

With the rapid advance of material science and technology, porous materials are of importance in engineering and industry owing to their excellent high temperature resistance, together with light weight and high radiation attenuation coefficient. In particular, with rapid development of aerospace industry, porous materials used for thermal protection system (TPS) have attracted tremendous attention and wide research interest in actual engineering applications. Also, as the porous materials often have periodical microstructure and materials coefficients vary rapidly in small reference cells, it is essential to develop a new effective and accurate numerical method for predicting the physical and mechanical performance of the porous materials.

Heat transfer problem arising from porous materials will be discussed in this paper. It involves materials with a large number of holes. Under such conditions, the direct accurate numerical computation of the solution is difficult because it would require a very fine mesh, and thus a prohibitive amount of computation time, even for the supercomputer. An effective way to overcome this difficulty is to develop homogenization method and the corresponding multiscale algorithms that can

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be used to obtain the equivalent material parameters as well as calculate the actual temperature fields in a microscale from the macroscopic responses, which cannot only save the computing resources but also ensure the computational accuracy [1–5]. However, the results of the multiscale methods for the heat conduction equation in a porous domain containing many holes are very limited. Bakhvalov [2] introduced the asymptotic homogenization method for the solutions of those problems in porous materials. On this basis, Cui et al. [3–5] studied the coupled conduction and radiation problem with rapidly oscillating coefficients, and obtained a high-order expansion for the problem. Allaire and El Ganaoui [6] and Ma and Cui [7] discussed the heat conduction model with ε^{-1}-order non-linear radiation boundary conditions by homogenization method, and justified the convergence. Cao [8] presented the multiscale asymptotic expansions of the solutions for the elliptic equation in a perforated domain. As for solving the heat conduction problem of porous materials with Neumann boundary conditions on the surfaces of pores, these pores are filled with an almost degenerated materials, which is called the hole-filling method [9,10]. Actually, engineers and scientists usually utilize the hole-filling method to predict the macroscopic performance of porous materials. From the viewpoint of physics, as the material parameters of the weak phase go to zero, the limit procedure is obvious. However, a rigorous theoretical justification has not been found in the available literature. Inspired by the idea reported in [9,10], in this work, the hole-filling method is proposed through filling all holes with a compliant material into the heat conduction equation in a porous domain. Also, as the material parameters of the weak phase go to zero, the mathematical proof of the limit procedure is clearly given. It should be noticed that we cannot directly follow the same idea presented in [9,10] to prove the error estimates of the hole-filling method for the heat conduction equation owing to the existence of the time derivative term.

Based on the traditional homogenization method [11–13], various multiscale approaches for periodic problems have been proposed, refer to Refs. [6,14–18]. However, they only considered the homogenization results and first order asymptotic expansions. In some cases [3–5,7–10], the homogenized solutions and first-order solutions are not enough to capture the local fluctuation in some physical and mechanical fields, hence it is necessary to find higher-order multiscale asymptotic expansions for the solutions. Allaire et al. [19] treated a linear heat conduction problem with convection boundary condition using the second-order asymptotic expansion. Cao and coworkers [8–10,20] introduced second order correctors in periodic homogenization applied to the composite materials with small parameter ε, and derived the convergence results. Cui et al. [3–5,21] proposed a high-order multiscale method to predict the physical and mechanical properties of composite materials, and solved some practical engineering problems. By high-order correctors, the microscopic fluctuation of physical and mechanical behaviors inside the material can be acquired more accurately. As a matter of fact, in engineering computation, the period ε is a fixed smaller constant, not tends to zero. If substituting the first-order multiscale solution into original equation, one can find out that the residual is $O(1)$ even though H^1-norm of its error is $O(\varepsilon^{1/2})$ [3–5,7–10,21]. The local error $O(1)$ is not accepted for engineers who want to capture the local behavior inside materials. Therefore, it is very necessary to seek higher-order asymptotic method in real applications.

In this paper, we will mainly discuss the heat transfer behavior of periodic porous materials. On the basis of hole-filling method and homogenization method, this paper is to establish a novel second-order multiscale method through filling all holes with almost degenerated materials. We introduce correction terms into the first-order multiscale expansion of the temperature fields, define a family of cell functions, and propose a full mathematical justification for this limiting process. Finally, some significant examples are computed to show the accuracy and efficiency of the method developed.

The remainder of this paper is outlined as follows. Section 2 describes the heat conduction and the microstructure of porous materials in detail. In Section 3, we introduce the hole-filling method, and obtain the error estimates for this method. In Section 4, the higher-order multiscale asymptotic expansions for the original problem in a porous domain and the associated multiphase problem in a domain without holes are discussed, respectively. Also, the theoretical proof for the hole-filling method is obtained in detail. Finally, some numerical results are given, which validate the theoretical results reported in this paper.

Throughout the paper the Einstein summation convention on repeated indices is adopted. C denotes a positive constant independent of $\varepsilon,\ \delta$.

2. Governing equations of heat conduction problem

In this section, the transient heat transfer problem of periodic porous materials is described in detail.

Let $Y = \left\{y : 0 \le y_j \le 1, j = 1, 2, 3\right\}$ and ω be an unbounded domain of R^3 which satisfies following conditions:

(B1) ω is a smooth unbounded domain of R^3 with a 1-periodic structure.
(B2) The cell of periodicity $Y^* = \omega \bigcap Y$ is a domain with a Lipschitz boundary, where Y^* is a reference periodicity cell, shown in Fig. 1(b).
(B3) The set $Y \backslash \bar{\omega}$ and the intersection of $Y \backslash \bar{\omega}$ with the δ_0 neighborhood of ∂Y consist of finite number of Lipschitz domains separated from each other and from the edges of the cube Y by a positive distance.
(B4) The cavities are convex.

Then, the domain Ω^ε, as shown in Fig. 1(a), has the form: $\Omega^\varepsilon = \Omega \bigcap \varepsilon\omega$, where Ω is a bounded Lipschitz convex domain of R^3. Similarly to the definitions in [4,8–10], let Ω_0^ε be a subdomain of the whole domain $\bar{\Omega}^\varepsilon = \bar{\Omega}_0^\varepsilon \bigcup (\bar{\Omega} \backslash \Omega_0)$, consisting of the union of periodic cells, i.e. $\bar{\Omega}_0 = \bigcup_{z\in\hat{T}_\varepsilon} \varepsilon(z + \bar{Y})$, $\bar{\Omega}_0^\varepsilon = \bar{\Omega}_0 \bigcap \varepsilon\bar{\omega}$ is shown in Fig. 1(a), where $\hat{T}_\varepsilon$ is the subset of Z^n consisting of all z, such that $\varepsilon(z + Y) \subset \Omega$, $\mathrm{dist}(\partial\Omega_0, \partial\Omega) \ge \varepsilon/2$, $\Omega_1 = \Omega \backslash \bar{\Omega}_0$. They are illustrated in Fig. 1.