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A multilevel sampling method for detecting sources in a stratified ocean waveguide[☆]

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ABSTRACT

In the reconstruction process of sound waves in a 3D stratified waveguide, a key technique is to effectively reduce the huge computational demand. In this work, we propose an efficient and simple multilevel reconstruction method to locate the accurate position of a point source in a stratified ocean. The proposed method can be viewed as a direct sampling method since no solutions of optimizations or linear systems are involved. The novel method exhibits several strengths: fast convergence, robustness against noise, advantages in computational complexity and applicability for a very small number of receivers.

1. Introduction

The primal goal of this work is to develop an effective numerical method for locating underwater sound sources in a stratified ocean environment. Sound propagation in the ocean environment has great importance, and a large number of experiments have been carried out for long-range propagations. Many numerical schemes have been proposed for range-dependent propagations and scattering problems in the ocean environment [1–3]. Mathematical investigations and modeling of wave propagations in underwater environment are widely available in literature; see [4–13] and the references therein.

Underwater sound propagation has been studied based on the physical principles of the acoustics. As a simple but reasonably realistic model for studying the effect of the underwater sound wave propagation we shall consider the stratified media [4,5] in this work. To simplify the configuration but still retain the physical features of interest, we model the problem as a horizontally stratified waveguide, and assume the sound speed c depends essentially on the depth of the waveguide. A typical sound velocity profile consists of the surface channel, the thermocline and isothermal layers; see Fig. 1 [14]. As we know, the surface channel is mainly formed due to a shallow isothermal layer appearing in winter, but can be caused also by an input of fresh water close to river estuaries, or by water which is quite cold in surface. Actually, a homogeneous layer (surface mixed layer, with a refractive index $n_1(z)$) of constant sound velocity is often present in the first few meters. It

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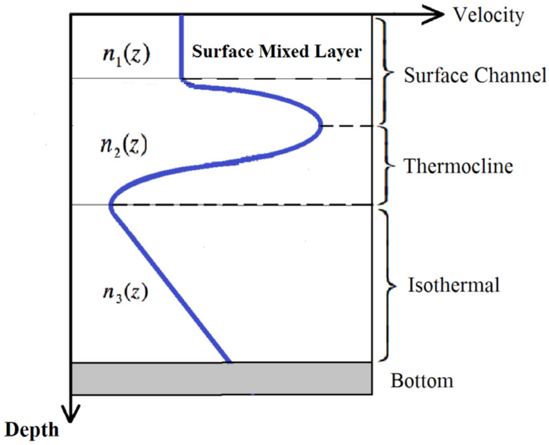

Fig. 1. The demonstration of a typical sound velocity profile.

corresponds to the mixing of superficial water through surface agitation. When the temperature does not vary with depth, the sound velocity increases linearly with depth because of the hydrostatic pressure effect. And this part is the rest of the surface channel that has $n_2(z)$ as its refractive index. The thermocline is a layer in which both temperature and sound velocity decrease with depth, while the isothermal layer is a layer with constant temperature, where the sound speed increases linearly with depth due to the hydrostatic pressure.

In stratified media, sound waves can be trapped by acoustic ducts to propagate horizontally [5,13], therefore the scattering of sound waves by bounded obstacles is much more complicated than that in homogeneous media.

The wave propagation in stratified media has been well studied, and we have a general understanding and a detailed description of how sound travels in the ocean. The theory could be applied to make quantitative computations of the sound field induced by an artificial source. Some mathematical modeling was studied for the wave propagation in stratified media [8,15,9], several methods were developed for solving the modeling problem and very interesting results were observed [16,17,10]. Other studies can be found in [18–23].

However, contrary to the large number of techniques developed for simulating wave propagations, few studies exist for inverse underwater sound problems in three dimensions, and nearly all existing computations are for the reconstruction of 2D obstacles. Imaging a scatterer in a waveguide is much more challenging than in the free space. As mentioned in [24], due to the presence of two parallel infinite boundaries of the waveguide, only a finite number of wave modes can propagate at long distance, while the other modes decay exponentially [25]. Even though many methods have been developed for inverse acoustic scattering problems, e.g., the MUSIC-type algorithm to locate small inclusions [26], the generalized dual space method [27], the linear sampling method [28–30], the Kirchhoff migration-based method [31], and the direct sampling method [32,33] to reconstruct obstacles, they are all for acoustic waves in homogeneous waveguides. More investigations are desired for inverse problems in 3D stratified waveguides.

The detection of black boxes of airplanes attracts much more attention recently since the air crash of MH370. A multilevel sampling method is proposed in this work to detect the location of a source in a stratified ocean waveguide with local non-stratified perturbations, which would provide an effective and simple alternative to estimate the positions of the black boxes. We shall study a full 3D underwater stratified model, present a mathematical analysis and an effective algorithm to locate the sound source in the complicate stratified ocean environment. A time-harmonic point source is assumed and located in the essentially stratified ocean waveguide, which is bounded above by a horizontal planar free surface where the acoustic pressure p vanishes, i.e., $p = 0$, and below by a horizontal planar bottom on which the normal derivative of the acoustic pressure p vanishes, namely, $\frac{\partial p}{\partial \nu} = 0$. This set-up and its studies can be found in the lecture notes of physics [11] and many other works, see, e.g., [9,17,10,12,33]. Our new method can be applied to other stratified oceans with appropriate modifications.

The paper is organized as follows. In Section 2, the acoustic model and Green's function of the 3D three-layered waveguide are stated, along with some useful notation, properties and identities. In Section 3, we formulate the direct scattering problem of 3D three-layered waveguide with a known inclusion, and an iterative method is proposed to solve the direct scattering problem of the perturbed waveguide. Section 4 describes a novel multilevel sampling method using partial scattered data, and Section 5 provides extensive numerical experiments to evaluate the performance of the multilevel sampling method. Finally, some concluding remarks are presented in Section 6.

2. Acoustic model in a three-layered waveguide and Green's function

In this section, we describe the direct scattering problem of our interest. Consider a three dimensional waveguide Ω in $\mathbb{R}^3_h = \mathbb{R}^2 \times (0, h)$, where $h > 0$ is the depth of the ocean. The third coordinate axis is singled out as the one orthogonal to