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A note on a discrete time MAP risk model

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ABSTRACT

In this paper, we use a discrete time Markov additive process to model the surplus process for an insurance company. Assume that the interclaim times and the claim sizes are both regulated by an underlying Markov chain. We present a recursive formula for the Gerber–Shiu function by two methods. Some numerical examples are also given to show the solution procedure.

1. Introduction

The continuous time Markov additive process (abbreviated as MAP in this paper) is a very popular model to describe the surplus process of an insurance portfolio as it includes the classical compound Poisson model (e.g. [1, Section IV]), the renewal risk process with phase-type inter-arrival times [2,3], the Markov-modulated risk process (e.g. [4,5]), and the semi-Markovian model proposed by Albrecher and Boxma [6]. Recently, there are a lot of contributions to the MAP risk model. For example, [7,8] studied some generalized discounted penalty functions in the MAP risk model; whereas Zhang et al. [9] investigated the absolute ruin problem under debit interest force. Cheung and Feng [10] proposed a more general Gerber–Shiu discounted function in the MAP risk model. Zhang and Cheung [11] studied the Gerber–Shiu function and the expected discounted dividend payments in a continuous time MAP risk model under a periodic dividend strategy.

Motivated by the successful application of the continuous time MAP in risk theory, in this paper we study the ruin problems in a discrete time MAP risk model. We remark that some contributions to discrete time risk models controlled by Markov chain have been made by some actuarial researchers. For example, [12] proposed a compound Markov binomial risk model and studied the ruin probabilities; whereas Yuen and Guo [13] studied the Gerber–Shiu function in the same model. Cossette et al. [14] proposed a binomial risk model in a Markovian environment and investigated the ruin probabilities. Yuen and Guo [15] and Xiao and Guo [21] considered a compound binomial risk model with delayed claims, which is in fact a discrete time risk model with Markovian type dependence between successive claim sizes. Yang and Zhang [16] studied the discounted joint distribution of the surplus before ruin and the deficit at ruin in a discrete Markov risk model. Recently, Chen et al. [17] study the expected discounted dividend in a semi-Markov risk model and Chen et al. [18] study the survival probabilities in a semi-Markov risk model. Note that the main results of these two papers are obtained in the two(or three)-state model. When the Markov process has a high dimensional state space, their arguments will become very complicated.

We note that many discrete time risk models with dependence structure can be described via a discrete time Markov process. The main work of this paper is to introduce a more general discrete time risk model with Markovian type dependence structure, which includes many well known models as special cases. We modify the compound binomial risk model by assuming that the claim occurrence and the claim size are dependent on the states before and after transition of

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a Markov chain and call this general model a discrete time MAP risk model. The main contribution is to provide a recursive algorithm for computing the Gerber–Shiu function. Different from the previous works in the literature, the algorithm is described in terms of matrix functions, so that it can be applied to computing the ruin functions when the Markov state space is finite. It is the first time to derive matrix recursive formula for computing the Gerber–Shiu functions.

Throughout of this paper, let $\mathbb{N} = \{0, 1, 2, \dots, \}$ and $\mathbb{N}^+ = \{1, 2, \dots\}$ be the nonnegative integers set and strictly positive integers set, respectively. Let $J = \{J_t\}_{t\in\mathbb{N}}$ be a discrete time Markov chain with finite state space $\mathcal{E} = \{1, 2, \dots, m\}$. Assume that J is homogeneous and irreducible. Denote by $\boldsymbol{\pi}^T = (\pi_1, \dots, \pi_m)$ the stationary distribution row vector of J, and let $\mathbf{P}_0 + \mathbf{P}_1$ be the one-step transition probability matrix, where $\mathbf{P}_0 = [p_{0,ij}]_{i,j=1}^m$, $\mathbf{P}_1 = [p_{1,ij}]_{i,j=1}^m$ are two strictly substochastic matrices denoting respectively the probabilities of state transitions with and without an accompanying claim. Furthermore, assume that the claim size distribution is dependent on the Markovian states of J immediately before and after the state transition. Whenever the transition from state i to state j occurs and a claim arrives, the claim size has probability mass function f_{ij}, mean μ_{ij} and probability generating function $\hat{f}_{ij}(z) = \sum_{x=1}^{\infty} z^x f_{ij}(x)$. More specifically, for $i, j \in \mathcal{E}$ we have

$$\begin{cases} \mathbb{P}(X = 0, J_t = j | J_{t-1} = i) = p_{0,ij}, \\ \mathbb{P}(X = x, J_t = j | J_{t-1} = i) = p_{1,ij} f_{ij}(x), \quad x \in \mathbb{N}^+, \end{cases} \tag{1.1}$$

where X is the generic variable of the claim size. Given the initial surplus $u \in \mathbb{N}$, we define the discrete time MAP risk process by

$$U_t = u + t - S_t, \quad t \in \mathbb{N}, \tag{1.2}$$

where $S_t = \sum_{i=1}^t X_i$ (with $S_0 = 0$) is the aggregate claims process. The bivariate process (J, S) is a discrete time MAP.

For notational convenience, let

$$\mathbb{E}_i(\cdot) = \mathbb{E}(\cdot | J_0 = i), \qquad \mathbb{P}_i = \mathbb{P}(\cdot | J_0 = i).$$

For two matrices $\mathbf{A} = [a_{ij}]_{i,j=1}^m$ and $\mathbf{B} = [b_{ij}]_{i,j=1}^m$ with the same dimension, $\mathbf{A} \circ \mathbf{B}$ means entry-wise multiplication, i.e. $\mathbf{A} \circ \mathbf{B} = [a_{ij} b_{ij}]_{i,j=1}^m$. Whereas for a matrix $\mathbf{A}$, we denote by $[\mathbf{A}]_{ij}$ the (i, j)th entry.

Let $\hat{\mathbf{f}}(z) = [\hat{f}_{ij}(z)]_{i,j=1}^m$. Using the same arguments as in Proposition 2.1 in Section XI of [19], we know that the (i, j)th entry in the matrix-valued generating function for S_t is given by

$$\mathbb{E}_i[z^{S_t}; J_t = j] = [\mathbf{K}(z)^t]_{ij}, \tag{1.3}$$

where

$$\mathbf{K}(z) = \mathbf{P}_0 + \mathbf{P}_1 \circ \hat{\mathbf{f}}(z), \quad 0 < z \le 1. \tag{1.4}$$

Note that by analytic extension $\mathbf{K}(z)$ is well defined for all z in the unit circle. By the Perron–Frobenius theorem, the spectral radius of $\mathbf{K}(z)$ for $0 < z \le 1$, denoted by $\kappa(z)$, is strictly positive and simple. By Corollary 2.8 and 2.9 in Section XI of [19], we know that the following analytic drift holds

$$\lim_{t\to\infty} \frac{S_t}{t} = \kappa'(1) = \sum_{i=1}^m \sum_{j=1}^m \pi_i p_{1,ij} \mu_{ij}, \tag{1.5}$$

and it does not depend on the initial Markovian state. Thus, to ensure that U_t has a positive drift, we assume that the following net profit condition holds

$$\sum_{i=1}^m \sum_{j=1}^m \pi_i p_{1,ij} \mu_{ij} < 1. \tag{1.6}$$

Associated with the risk process U_t, we define the ruin time by $\tau = \inf\{t \in \mathbb{N}^+ : U_t < 0\}$ with $\tau = \infty$ if $U_t \ge 0$ for all $t \in \mathbb{N}^+$. In this paper, we are interested in the Gerber–Shiu function defined as (given initial state $i \in \mathcal{E}$ and initial surplus $u \in \mathbb{N}$)

$$\Phi_{ij}(u) = \mathbb{E}_i[v^\tau w(U_{\tau-1}, |U_\tau|) \mathbf{1}_{(\tau<\infty, J_\tau=j)} | U_0 = u], \quad j \in \mathcal{E}, \tag{1.7}$$

where $0 < v \le 1$ can be interpreted as discount factor, $\mathbf{1}_{(A)}$ is the indicator function of an event A, and $w(x_1, x_2)$ defined on $\mathbb{N} \times \mathbb{N}^+$ is a nonnegative bounded function. We set $w(x_1, x_2) = 0$ for $(x_1, x_2) \notin \mathbb{N} \times \mathbb{N}^+$. Write $\mathbf{\Phi}(u) = [\Phi_{ij}(u)]_{i,j=1}^m$.

The main goal of this paper is to present a recursive formula to calculate the Gerber–Shiu function in the discrete time MAP risk model. The rest of this paper is organized as follows. In Section 2, we present some preliminaries, among which a matrix, say $\mathbf{V}_v$, plays an important role in the analysis of the Gerber–Shiu function. The main results are given in Section 3, where we derive a recursive formula satisfied by the Gerber–Shiu function by two methods: one is based on the analytic arguments, while the other one is based on probabilistic arguments. Finally, in Section 4, we give some numerical examples to show that our approach is really applicable.