Journal of Computational and Applied Mathematics 309 (2017) 230–243

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Numerical study of signal propagation in corrugated coaxial cables

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ARTICLE INFO

Article history:
Received 29 February 2016
Received in revised form 30 June 2016

Keywords:
Maxwell's equations
FDTD method
Discontinuous Galerkin method
Coaxial cable

ABSTRACT

This paper is concerned with high-fidelity modeling of signal propagation in corrugated coaxial cables. Taking advantage of the axisymmetry, we reduce the 3-D problem to a 2-D problem by solving a time dependent Maxwell's equations in cylindrical coordinates. We develop a nodal discontinuous Galerkin method for solving our model equations. Stability and error analysis are proved for the semi-discrete scheme. Extensive numerical results are provided to demonstrate that our algorithm not only converges as our theoretical analysis predicts, but also is very effective in solving various signal propagation problems in practical corrugated coaxial cables.

1. Introduction

Due to the long standing and widespread usage of coaxial cables, there are many published papers on modeling wave and signal propagation through coaxial cables. Various methods [1,2], ranging from using experimental data to mathematical models, have been developed for transmission lines. For coaxial cables the two most common methods of mathematically modeling signal and wave propagation through the cables are to solve either the telegrapher's equations [3] (developed by Oliver Heaviside in the 1880s) or Maxwell's equations.

The telegrapher's equations treat the conductors in the coaxial cable as an infinite series of two-port elementary components, each representing an infinitesimally short segment of the transmission line. Each segment of the line is modeled by a circuit with four elementary components: a resistor and inductor in series, a shunt capacitor between the two conductors, and a shunt resistor between the two conductors [4]. The following telegrapher's equations are used to model the voltage V and current I of the transmitted signal on a transmission line with resistance R, inductance L, capacitance C, and conductance G:

$$\frac{\partial V}{\partial x}(x,t) = -L\frac{\partial I}{\partial t}(x,t) - RI(x,t),$$
$$\frac{\partial I}{\partial x}(x,t) = -C\frac{\partial V}{\partial t}(x,t) - GV(x,t).$$

Note that the telegrapher's equations are a coupled system of two one-dimensional partial differential equations (PDEs), making them quite simple and efficient to solve. However, since the telegrapher's equations are a one-dimensional representation of the coaxial cable, they do not take into account the geometry of the cable. Hence if the cable's cross

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http://dx.doi.org/10.1016/j.cam.2016.07.003

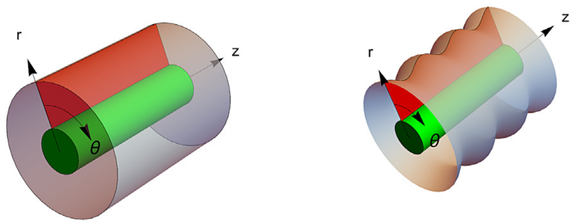

Fig. 1. (Left) A 3-D view of a coaxial cable. Red rectangle: cross-sectional domain; Green cylinder: inner conductor; Gray cylinder: outer conductor. (Right) A 3-D view of a corrugated coaxial cable. Red rectangle: cross-sectional domain. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

section changes at different locations such as the corrugated cable, then the effects of the corrugation cannot be accounted for without adding in an artificial term. In [5], Imperiale and Joly derived the telegrapher's model via an asymptotic analysis from 3-D Maxwell's equations for a lossy coaxial cable whose cross section is not homogeneous.

To account for the variable cross section cables, we resort to solving the Maxwell's equations in three-dimensional (3-D) space. However, this PDE system is much more complex and computationally intensive to solve than the telegrapher's equations. To reduce the computational cost and consider notions of the fact that many coaxial cables of interest have rotational symmetry about the z-axis (i.e. the angular component has no effect on the electric or magnetic fields), we often reduce the 3-D problem to a 2-D problem whose domain is the length-wise cross-section (the red part in Fig. 1).

Mathematical analysis of finite elements for axisymmetric Maxwell equations has been attracting an increasing interest since 2000. Ciarlet et al. initiated the study of axisymmetric Maxwell equations [6,7]. Later, in 2006, a least-squares method for axisymmetric div–curl systems was analyzed [8]. In that same time frame, multigrid methods were proposed and analyzed for axisymmetric Maxwell equations [9,10]. Subsequently, finite element methods were developed and analyzed for solving time-dependent axisymmetric eddy current models [11,12].

The goal of this paper is to explore the effect of corrugated coaxiable cable on the electric pulse propagation in more detail than others [13,14,5]. Here we estimate the effects of corrugation by solving Maxwell's equations in cylindrical coordinates to model the wave propagation between the two conductors of the corrugated coaxial cable. In [14], the nodal discontinuous Galerkin method (e.g., [15–17]) was extended to solve the 2-D cylindrical coordinate Maxwell equations. However, [14] does not provide any stability analysis nor error estimate of the method. Here, we first develop a similar method for our corrugated cable model, then we present a stability analysis and error estimate for the semi-discrete scheme. Finally, we use our algorithm to solve various corrugations and compared with the results obtained by the finite difference time domain (FDTD) method.

The rest of the paper is organized as follows. In Section 2, we present the axisymmetric Maxwell equations and show that the energy of the system is conserved. In Section 3, we introduce the nodal discontinuous Galerkin (nDG) method in both semi- and fully-discrete forms. Stability and convergence of the semi-discrete scheme is established rigorously. In Section 4, we present extensive numerical results verifying the theoretical analysis and applying the method to the wave propagation problem in various corrugated coaxial cables. Conclusions are in Section 5.

2. The governing equations

Replacing the curl operator in Cartesian coordinates by that in cylindrical coordinates (r, θ, z), we can easily obtain the Maxwell's equations in cylindrical coordinates (cf. [14]):

$$\frac{\partial E^r}{\partial t} - \frac{1}{r}\frac{\partial B^z}{\partial \theta} + \frac{\partial B^\theta}{\partial z} = 0 \tag{1}$$

$$\frac{\partial E^\theta}{\partial t} + \frac{\partial B^z}{\partial r} - \frac{\partial B^r}{\partial z} = 0 \tag{2}$$

$$\frac{\partial E^z}{\partial t} - \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rB^\theta\right) - \frac{\partial B^r}{\partial \theta}\right) = 0 \tag{3}$$

$$\frac{\partial B^r}{\partial t} + \frac{1}{r}\frac{\partial E^z}{\partial \theta} - \frac{\partial E^\theta}{\partial z} = 0 \tag{4}$$

$$\frac{\partial B^\theta}{\partial t} - \frac{\partial E^z}{\partial r} + \frac{\partial E^r}{\partial z} = 0 \tag{5}$$

$$\frac{\partial B^z}{\partial t} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rE^\theta\right) - \frac{\partial E^r}{\partial \theta}\right) = 0, \tag{6}$$

where (E^r, E^θ, E^z) and (B^r, B^θ, B^z) denote the electric and magnetic fields, respectively. For simplicity, we assume that the permittivity and permeability both equal 1.