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New versions of iterative splitting methods for the momentum equation☆

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ABSTRACT

In this paper we propose some modifications in the schemes for the iterative splitting techniques defined in Geiser (2009) for partial differential equations and introduce the parallel version of these modified algorithms. Theoretical results related to the order of the iterative splitting for these schemes are obtained. In the numerical experiments we compare the obtained results by applying iterative methods to approximate the solutions of the nonlinear systems obtained from the discretization of the splitting techniques to the mixed convection–diffusion Burgers' equation and a momentum equation that models a viscous flow. The differential equations in each splitting interval are solved by the back-Euler–Newton algorithm using sparse matrices.

1. Introduction

Nowadays, iterative splitting methods are considered as excellent decomposition methods to obtain higher-order results and to embed nonlinearities. This is due to the advantage of this technique in combining iterative and splitting behavior for decoupling physics problems. In this paper, we develop new nonlinear solvers that are modifications of the iterative splitting schemes defined in [1,2].

Iterative splitting schemes are used to solve nonlinear systems obtained from ordinary differential equations or spatially discretized partial differential equations. For their original scheme, one applies a Picard-iterative technique to solve the nonlinear systems, see [2,3]. The drawback is that such Picard's technique is a first order scheme, see [4]. Our benefit is embedding a nonlinear scheme, Newton's method, in the splitting methods. Such novel schemes are more accurate, higher order and accelerate the solver schemes.

The novelty consists in a modification based on the idea of using the solution of first equation of the split problem for updating operators in the second equation of the same iteration. We consider all possible variations for combining the nonlinear operators, getting six different schemes. We also derive parallel versions of such schemes in order to obtain

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additional faster schemes and an up to date nonlinear solver for large-scale computations. Further, we derive parallel versions of such schemes to obtain additional faster schemes and an up to date nonlinear solver for large scale computations, see [5].

The outline of the paper is as follows. In Section 2 we introduce our mathematical model. The splitting techniques are presented in Section 3. In Section 4, we discuss the theoretical results of the methods. Section 5 explains how to apply nonlinear solvers to the equations of the splitting and Section 6 applies the new splitting methods to a scalar and a vector valued PDE and compares the results with the ones of some classical splitting methods.

2. Mathematical model

A great variety of natural phenomena can be described by an ordinary differential equation or a partial differential equation, the solution of which not always can be obtained by analytical methods. In fact, in the majority of cases, it is much more practical using numerical methods in order to approximate the solution.

In the present paper we concentrate on partial differential equations given as:

$$\frac{du(v,t)}{dt} = f(t,u(v,t)), \quad u:\Omega\times\mathbb{R}\longrightarrow\mathbb{R},\ \Omega\subseteq\mathbb{R}^n$$
$$\text{Boundary condition}: u(v,t) = w(v,t), \quad (v,t)\in\partial\Omega\times[0,T]$$
$$\text{Initial condition}: u(v,0) = u_0(v), \quad v\in\Omega.$$

In case $n = 0$, we have an ordinary differential equation:

$$\frac{du(t)}{dt} = f(t,u(t)), \quad t\in[0,T],\ u:[0,T]\longrightarrow\mathbb{R},$$

and in case $n > 0$, we will obtain a system of ordinary differential equations.

This is the case for convection–diffusion–reaction-equations, see [6–9], and for a computational simulation of heat-transfer [10], which, with the above notation, is a particular case with $n = 2$. Function $f(t, u(x, y, t))$ can contain partial derivatives up to second order u_x, u_y, u_{xx} and u_{yy}.

Specifically, we deal with a particular form for function $f(t, u(v, t))$, when it can be expressed in the following form:

$$\frac{du(v,t)}{dt} = A(u(v,t))u(v,t) + B(u(v,t))u(v,t) + g(v,t), \tag{1}$$

with $t \in [0, T]$, the initial condition is known $u(v, 0) = u_0(v)$, and $A(u), B(u)$ are operators in a Banach space X involving only spatial derivatives of u, whereas $g(v, t)$ is an exterior perturbation.

3. Splitting techniques

Splitting techniques can be used with the aim of decomposing the original problem into a sequence of simpler problems when the size of the problem is big or maybe if we need to solve the problem taking into account physical properties of some parts of the equation. Sometimes, we want to separate the nonlinear part of the equation from the linear part or just our aim is using different numerical methods in each part of the equation, always with the final objective of building efficient methods with the usual properties of accuracy and stability.

The traditional method is the sequential operator splitting, but nowadays iterative splitting is being the objective of different studies, [11,12]. In all cases, we discretize the time interval $[0, T]$ in N subintervals by means of the partition $0 < t^1 < t^2 < \cdots < t^n < t^{n+1} < \cdots < T$ and solve a different problem consecutively in each of these subintervals.

3.1. Classical operator splitting techniques

Between the different splitting algorithms traditionally used, let us mention the following ones

3.1.1. Sequential operator splitting

In this scheme we solve two sub-problems in $[t^n, t^{n+1}]$ sequentially connected via the initial conditions. First we solve the problem considering only the first operator with the given initial condition and after that, the problem is solved considering the second operator with initial condition the solution obtained in the first problem, that is:

$$\begin{aligned} \frac{d\tilde{u}(t)}{dt} &= A(\tilde{u}(t))\tilde{u}(t), \qquad \tilde{u}(t^n) = u(t^n) \\ \frac{du(t)}{dt} &= B(u(t))u(t), \qquad u(t^n) = \tilde{u}(t^{n+1}), \end{aligned} \tag{2}$$